SOME OPTIMUM PROPERTIES OF RANKING PROCEDURES[1]

By Morris L. Eaton[2]

Stanford University

1. Introduction. The purpose of this paper is to present a fairly general treatment of a class of statistical problems commonly referred to as ranking (or selection) problems. The presentation is decision theoretic and attention is restricted to the symmetric case so that certain invariance arguments are applicable. For the problem of selecting the best one of several populations, Bahadur (1950) and Bahadur and Goodman (1952) have proved that, for certain families of distributions, the natural selection procedure uniformly minimizes the risk among all symmetric procedures for a large class of loss functions. More recently, Lehmann (1966) has given an alternative proof of the above result and has indicated several other optimum properties of the natural selection procedure. (The results presented here were obtained by the author (1966) independently of the results given by Lehmann (1966).)

In Section 2, we introduce a monotonicity property (called property M) for density functions of k real variables and k real parameters. The class of densities with property M includes the class of densities considered by Lehmann (1966). This class also contains densities of practical interest which have not been considered in previous treatments of ranking problems.

The general ranking problem which was explicitly described by Bechhofer (1954) is the following: on the basis of a set of observations, we wish to partition the set of coordinate values of a k-dimensional parameter vector $\theta = (\theta_1, \cdots, \theta_k)$ into s disjoint subsets, say $\lambda_1, \cdots, \lambda_s$, such that λ_1 contains the k_1 largest components of θ, λ_2 contains the k_2 next largest components of $\theta, \cdots$, and λ_s contains the k_s smallest components of θ where $1 \leqq k_i < k$ and $\sum k_i = k$. In Section 3, we discuss the assumptions on the set of observations and introduce the loss structure. In Section 4, it is shown that if the density of the observations has property M then the natural selection procedure for the problem above: (i) is Bayes for every prior distribution which is symmetric in θ, (ii) uniformly minimizes the risk among symmetric decision rules, and (iii) is minimax and admissible.

Section 5 is devoted to showing that, in a certain sense, property M is a natural assumption for symmetric selection problems. More specifically it is shown, under regularity conditions, that if the natural selection procedure is uniformly best among symmetric decision rules, then the underlying density has property M. In Section 6, certain results on most economical decision rules

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[2] Now at The University of Chicago.