MULTISTAGE SAMPLING PROCEDURES BASED ON PRIOR DISTRIBUTIONS AND COSTS

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1. Introduction. The subject of this investigation is a problem of quality control: An article is produced in large lots; by means of a sampling procedure, distinguishing between "effective" and "defective" items, the decision upon acceptance or rejection of the lot is made.

The most general form of such a sampling procedure is a multistage test with variable sample sizes. The maximum number of stages (k) may be any integer between 1 and the lot size (N). Then the k-stage sampling procedure consists of the following instructions: Firstly, a fixed number between 0 and N is determined to be the size of the first sample. Secondly, for each stage j $(j = 1, \cdots, k - 1)$ of the procedure, the size of the $(j + 1)$st sample, as a function of the outcome of the first j samples, is given. Thirdly, for the final stage k and for all those cases where on stage j the size of the $(j + 1)$st sample is zero, it is prescribed (in terms of the sampling outcome thus far obtained) whether the lot has to be accepted or rejected. If all sample sizes are fixed in advance, then it is merely necessary to determine whether the procedure on stage j is to be continued and, if not, what terminal decision is to be made.

We intend to take economic considerations as a basis for the construction of such sampling procedures. We thus assume the costs and the prior distribution of the number of defective items in the lot to be given. Then among all k-stage sampling procedures the one with the lowest total expected costs is considered optimal.

2. Optimal k-stage sampling procedures. To the ith item in a lot of size N the random variable x_i $(i = 1, \cdots, N)$ is assigned, asuming the value 0 or 1 if the item is effective or defective, respectively. Then the sampling space is

$$S = \{x = (x_1, \cdots, x_N): x_i = 0 \text{ or } 1, i = 1, \cdots, N\}. \tag{1}$$

More generally, S can be thought of as the cartesian product of some sets $S_1, \cdots, S_N$, i.e. $S = S_1 \times \cdots \times S_N$ and $x = (x_1, \cdots, x_N)$, with $x_i \,\varepsilon\, S_i$ $(i = 1, \cdots, N)$. Let F be the σ-field of events over S, let F_i be the σ-field of events which can be described by $(x_1, \cdots, x_i)$, and let P be a probability measure over F. Then $F_0 \subset F_1 \subset \cdots \subset F_N$ with $F_0 = \{\varnothing, S\}$ and $F_N = F$. Let the symbol E_i denote the conditional expectation of a random variable depending on x, given F_i $(i = 0, 1, \cdots, N)$. Thus E_0 or E means the unconditional expectation.

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