ON SELECTING A SUBSET OF k POPULATIONS CONTAINING THE BEST[1]

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1. Introduction. This paper is concerned with the problem of selecting a subset of k populations which is in some sense an optimal subset. In the usual subset selection type setup we are given k populations $\pi_1, \pi_2, \cdots, \pi_k$ with densities $f_{\theta_1}, f_{\theta_2}, \cdots, f_{\theta_k}$. In general the parameters θ_i are not known and usually range over some subset of the real line. For convenience it is assumed that the larger the parameter θ, the more preferable is the selection of the corresponding population. The population with the largest parameter is called the best and a selection of any subset containing the best population is called a correct selection. If the selection proceeds according to some rule R, then the subset selected is required to contain the best population with a specified probability γ.

For many types of densities $f_\theta(x)$, e.g. normal, gamma, binomial and more general situations, different types of rules have been proposed and numerous properties of the rules have been investigated. References to some of the literature on the subject can be found in Gupta (1965).

In this paper we first investigate the more elementary problem of defining "optimal" subset selection rules for the case where we have k *fixed* density functions and only the correct pairing of the densities and populations is unknown.

Section 2 contains a decision theoretic formulation of the selection problem and a solution is obtained under the usual symmetry conditions. In Section 3 we examine the solution for the exponential family when the parameters are in a "slippage" type configuration. In Section 4 we obtain a result for the normal case where the parameters are permitted to vary. Section 5 contains two remarks; one concerning scale invariant procedures, the other concerning the fact that certain "classical" procedures appear as limits of the procedures obtained in Section 3 for the exponential family.

The results presented below are all quite elementary, however, it is hoped that the selection procedures introduced (especially those in Theorem 3.1) will have some interest as alternatives to those procedures now in use. Theorem 4.1 indicates that the usual procedures are not the "best" for certain specific situations.

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