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ON TESTS OF THE EQUALITY OF TWO COVARIANCE MATRICES

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0. Introduction. Let $X = (X_1, \cdots, X_p)'$, $Y = (Y_1, \cdots, Y_p)'$ be independently and normally distributed column vectors with unknown means $\xi = (\xi_1, \cdots, \xi_p)'$, $\eta = (\eta_1, \cdots, \eta_p)'$ and unknown positive definite covariance matrices Σ_1, Σ_2 respectively. We are interested here to test the null hypothesis $H_0 : \Sigma_1 = \Sigma_2$. This problem remains invariant under the group G of affine transformations (linear transformations together with translations) of the form $X \to AX + b_1$, $Y \to AY + b_2$ where A is a $p \times p$ non-singular matrix and b_1, b_2 are p-dimensional column vectors. Let $X_1, \cdots, X_{N_1}$ be the samples of sizes N_1 and N_2 from X, Y respectively. Writing

$$\bar{X} = \textstyle\sum_1^{N_1} X_i/N_1, \qquad \bar{Y} = \sum_1^{N_2} Y_i/N_2,$$

$$S_1 = \textstyle\sum_1^{N_1} (X_i - \bar{X})(X_i - \bar{X})' \quad \text{and} \quad S_2 = \sum_1^{N_2} (Y_i - \bar{Y})(Y_i - \bar{Y})';$$

a set of maximal invariants in the sample space with respect to G (with sufficiency and invariance reduction in either order, see Hall, Wijsman and Ghosh (1965)), is $R_1, \cdots R_p$, the characteristic roots of $S_1S_2^{-1}$. The corresponding set of maximal invariants in the parametric space under G is $\theta_1, \cdots \theta_p$, the characteristic roots of $\Sigma_1\Sigma_2^{-1}$. In terms of maximal invariants our testing problem can be reduced to that of testing the null hypothesis:

$$H_0: \theta_1 = \cdots = \theta_p = 1 \tag{0.1}$$

We will consider here the following alternative.

$$H_1: \textstyle\sum_1^p \theta_i > p. \tag{0.2}$$

The dual alternative $\sum_1^p \theta_i < p$ is reduced to (0.2) by interchanging the roles of the X's and Y's.

For this problem several invariant tests are known to us (i) a test based on $|S_2|/|S_1|$, (ii) a test based on $\operatorname{tr} S_1S_2^{-1}$, (iii) Roy's test based on the largest and the smallest characteristic roots of $S_1S_2^{-1}$, (iv) Kiefer and Schwartz's test (1965) based on $|S_1 + S_2|/|S_2|$.

From Anderson and Das Gupta (1964) it follows that the power of each of the above tests for testing H_0 against H_1 is a monotonically increasing function of each θ_i. Kiefer and Schwartz's test is admissible for this problem. We will suggest here another test based on $\operatorname{tr} S_2(S_1 + S_2)^{-1}$ which is locally best invariant.

1. Locally best invariant test. Let $\mathfrak{X}$ be the space of maximal invariant R in the sample space and Ω be the space of corresponding maximal invariant θ in

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