The Annals of Mathematical Statistics
1968, Vol. 39, No. 3, 1086–1097

ABSTRACTS OF PAPERS

(*Abstracts of papers presented at the Central Regional meeting, East Lansing, Michigan, March 18–20, 1968.*)

2. Distribution of linear combination of order statistics from the rectangular population. M. M. Ali, University of Western Ontario.

Let $X_1 < X_2 < \cdots < X_n$ be the order statistics of a random sample of size n from the Rectangular (0, 1) population. We prove that $F_n(z) = \Pr\,(\sum_1^n 1_i X_i \leqq z)$ is given by $F_n(z) = \sum_{\nu=0}^n H(z, a_\nu, n)/P_\nu(a_\nu)$ for all z where $a_\nu = \sum_{i=\nu}^n 1_i$, $\nu = 1, \cdots, n$, and $a_0 = 0$, $P_\nu(x) = (a_0 - x)(a_1 - x) \cdots (a_n - x)/(a_\nu - x)$ and $H(x, a_\nu, n) = \{\frac{1}{2}(z - a_\nu) + \frac{1}{2}|z - a_\nu|\}^n$. When some of the a_ν's coincide $F_n(z)$ is still well defined by taking appropriate limit of the expression for $F_n(z)$. (Received 15 January 1968.)

3. Distribution-free tests for multivariate independence, symmetry and k-sample problems. C. B. Bell and Paul Smith, Case Western Reserve University.

Let H_0': $F(x_1, \cdots, x_s) = F(t(x_1, \cdots, x_s))$ for all permutations t; and H_0'': There exist G_i such that $F(x_1, \cdots, x_s) = \prod_1^m G_i(x_{r_i}, \cdots, x_{q_i})$, where $r_i = 1 + q_{i-1}$; and $z = (x_{11}, \cdots, x_{1s}; \cdots; x_{n1}, \cdots, x_{ns})$ be the generic data point. The permutation groups under which the likelihood functions are invariant are, respectively, S' of order $(n!)(s!)^n$ and S'' of order $n!(n!)^m$. Theorem 1. *A statistic is* DF wrt $H_0'[H_0'']$ iff *it is a measurable function of some permutation statistic based on* S' $[S'']$. *In either case, its null distribution is discrete with probabilities integral multiples of the reciprocal of the order of the permutation group.* Theorem 2. *Against a simple alternative the most powerful* DF *test is a permutation test based on the alternative likelihood function.* This test is also most powerful DF for a Koopman-Pitman class generated by the alternative. For the k-sample problem $z = (x_{111}, \cdots, x_{11s}; \cdots, x_{kn_ks})$ and H_0''': $F_1(x_1, \cdots, x_s) = \cdots = F_k(x_1, \cdots, x_s)$. The permutation group S''' is of order $(n_1 + \cdots + n_k)!$. Theorems analagous to Theorems 1 and 2 are valid here. (Received 2 February 1968.)

4. On the monotonicity of $E_p(S_t/t)$. Y. S. Chow and W. J. Studden, Purdue University.

Let $S_n = X_1 + \cdots + X_n$ be the sums of independent, identically distributed random variables X_n with $P[X_n = 1] = p$ and $P[X_n = 0] = q = 1 - p$. Let t be a stopping time relative to the sequence X_n. The following theorem was conjectured by H. Robbins: Theorem. *If* $P_p[t < \infty] = 1$ *for every* $0 < p < 1$, *then* $E_p(S_t/t) \leqq E_{p'}(S_t/t)$ *for* $0 < p \leqq p' < 1$. In the proof, the Wald's equation $E_pS_t = pE_pt$ for a bounded stopping time t has been utilized. The result holds when the X_i are iid with an exponential density $C(p)e^{Q(p)x}$ with respect to some measure $d\mu$ where $Q(p)$ is increasing in p. This includes the normal and poisson distributions. (Received 29 January 1968.)

5. Some remarks on Scheffé's solution to the Behrens-Fisher problem. Morris L. Eaton, University of Chicago.

Let $X_1, \cdots, X_m$ and $Y_1, \cdots, Y_n$ $(m \leqq n)$ be two independent random samples from $N(\mu_1, \sigma_1^2)$ and $N(\mu_2, \sigma_2^2)$ populations respectively. In this paper, it is shown that Scheffé's proposed solution (Scheffé, H. (1943), *Ann. Math. Statist.* **13** 371-388) to the problem of testing that $\mu_1 = \mu_2$ is equivalent to the following procedure: (i) form $\bar{X}$, $\bar{Y}$ and the joint