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THE GEOMETRY OF AN $r \times c$ CONTINGENCY TABLE[1]

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1. Introduction. Any contingency table can be normalized to have entries which add to one, and then all possible $r \times c$ two-way tables can be represented by points within the $(rc - 1)$-dimensional simplex

$$S_{rc} = \{(x_{11}, x_{12}, \cdots, x_{1c}; \cdots; x_{r1}, \cdots, x_{rc}) : x_{ij} \geqq 0, \textstyle\sum_{i,j} x_{ij} = 1\} \tag{1.1}$$

in rc-space. A deeper understanding of the geometry associated with this simplex might allow us to deal with the corresponding contingency tables in a more enlightened manner.

In a previous paper, [2], ideas about 2×2 contingency tables were discussed in terms of the geometry of the 3-dimensional simplex. Here we generalize these ideas and in particular we derive the loci of (a) all points corresponding to tables whose rows and columns are independent, (b) all points corresponding to tables with a given interaction structure, and (c) all points corresponding to a table with a fixed set of marginals. Finally we conclude with a discussion of the generalization of our results to multidimensional tables.

2. The simplex of reference. We examine the simplex S_{rc} by means of rc-dimensional barycentric co-ordinates, [1], and choose the simplex of reference (with vertices A_{ij} for $i = 1, 2, \cdots, r$ and $j = 1, 2, \cdots, c$) so that

$$\begin{aligned} A_{11} &= (1, 0, 0, \cdots, 0, 0, 0), \\ A_{12} &= (0, 1, 0, \cdots, 0, 0, 0), \\ &\vdots \\ A_{r(c-1)} &= (0, 0, 0, \cdots, 0, 1, 0) \\ A_{rc} &= (0, 0, 0, \cdots, 0, 0, 1) \end{aligned} \tag{2.1}$$

correspond respectively to the $r \times c$ tables

$$\begin{matrix} 1 & 0 & \cdots & 0 & 0 \\ 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & \cdots & 0 & 0, \end{matrix} \qquad \begin{matrix} 0 & 1 & \cdots & 0 & 0 \\ 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & \cdots & 0 & 0, \end{matrix}$$

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