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ON A CHARACTERIZATION OF SYMMETRIC STABLE PROCESSES WITH FINITE MEAN[1]

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1. Introduction. Laha [1] studied the characterization of symmetric stable laws through regression properties and he has proved the following theorem.

THEOREM 1.1. *Let X and Y be two independent nondegenerate random variables whose expectations exist and are zero. Suppose that a structure is given by $U = aX + bY$, $V = cX + dY$ with a, b, c, d different from zero and $ad \neq bc$. Then both X and Y have symmetric stable distributions with the same exponent $\lambda > 1$, if and only if,*

(i) *there exists a constant $\delta > 0$ such that the relation $E[V \mid U] = \beta U$ a.e. holds for all a such that $0 < |a| < \delta$, and*

(ii) $$\beta = (ca^{-1}\alpha_1 |a|^\lambda + db^{-1}\alpha_2 |b|^\lambda)(\alpha_1 |a|^\lambda + \alpha_2 |b|^\lambda)^{-1}$$

where α_1 and α_2 are the scale parameters of the distributions of X and Y respectively.

Our aim in this paper is to derive a similar result to characterize symmetric stable processes with finite mean function. While this paper was in preparation, we noticed that Lucaks [2] has given a different characterization of symmetric stable processes.

2. Definitions. We shall now present some definitions and some results concerning stochastic processes and stochastic integrals. Let T be any bounded interval. We shall take $T = [0, 1]$ unless otherwise stated.

A stochastic process $\{X(t), t \varepsilon T\}$ is said to be a homogeneous process with independent increments if the distribution of the increments $X(t + h) - X(t)$ depends only on h but is independent of t and if the increments over non-overlapping intervals are independent.

Let $\{X(t), t \varepsilon T\}$ be a homogeneous process with independent increments. Let $\varphi(u; h)$ denote the characteristic function of $X(t + h) - X(t)$. It is well known that $\varphi(u; h)$ is infinitely divisible and $\varphi(u; h) = [\varphi(u; 1)]^h$. It can be shown that the stochastic integral,

$$\int_0^1 a(t)\, dX(t), \tag{2.1}$$

can be defined in the sense of convergence in probability for a large class of functions $a(\cdot)$ on [0, 1] which includes the class of infinitely differentiable functions on [0, 1]. This can be done by defining (2.1) for simple functions $a(\cdot)$ in the obvious manner and then extending the definition to functions which can be approximated by simple functions uniformly.

Let $\{X(t), t \varepsilon T\}$ be a homogeneous process with independent increments and

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