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ASYMPTOTIC SHAPES FOR SEQUENTIAL TESTING OF TRUNCATION PARAMETERS[1]

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1. Introduction. In an earlier paper [2] an asymptotic property of the Bayes sequential testing regions was proved for exponential families. With c, the cost of an observation, tending to zero, the regions, scaled down by a factor of $-\log c$, were shown to approach a limiting region. The limiting region depends on the *a priori* distribution only through its support, and is easily and explicitly described in terms of a modified maximum likelihood statistic. In this paper these results are extended to families with *truncation parameters*, that is, parameters that govern the range of the random variables.

The result in [2] is obtained by: (1) bounding the Bayes regions within and without by *constant a posteriori risk regions*, and (2) studying the asymptotic behaviour of the latter. The first part of the result is easily extended beyond exponential families, and this has been done by Kiefer and Sacks [1]. The second part is extended to truncation parameter families in this paper (Theorem 2). In order to make the paper self-contained, a simple proof of the extended first part is included (Theorem 1). A number of examples conclude the paper.

2. Truncation parameters. In a family $P(\cdot, \eta, \theta)$ of distributions, θ is a *truncation parameter* if it is real-valued, and if there exists a random variable T such that for $\theta_1 > \theta_2$, $P(\cdot, \eta, \theta_2)$ is obtained from $P(\cdot, \eta, \theta_1)$ by conditioning on the event $\{T \leqq \theta_2\}$. The families we are concerned with here depend on a finite number of parameters, some of which are truncation parameters, with the rest, if any, appearing as exponential parameters. Such a family we call an *exponential truncation family* $P(\cdot, \theta_1, \cdots, \theta_t, \eta_1, \cdots, \eta_s)$, characterized as follows: there exist on $(\Omega, \mathcal{B})$ a measure μ and $t + s$ random variables $T_1, \cdots T_t, Y_1, \cdots, Y_s$ such that the density of $P(\cdot, \theta_1, \cdots, \theta_t, \eta_1, \cdots, \eta_s)$ with respect to μ is given by

$$\exp\{\langle \boldsymbol{\eta} \cdot \mathbf{Y}\rangle - b(\boldsymbol{\theta}, \boldsymbol{\eta})\} \quad \text{when} \quad T_k \leqq \theta_k \quad \text{for} \quad k = 1, \cdots, t$$

and

$$0 \qquad\qquad \text{otherwise.}$$

Here $\boldsymbol{\theta} = (\theta_1, \cdots, \theta_t)$, $\boldsymbol{\eta} = (\eta_1, \cdots, \eta_s)$, $\mathbf{Y} = (Y_1, \cdots, Y_s)$, $\langle \cdot \rangle$ stands for the dot product, and b is the real-valued function required to normalize the density. The *parameter space* Θ is the Borel set of all $(\boldsymbol{\theta}, \boldsymbol{\eta})$ for which such a b exists.

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