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PARTIALLY BALANCED INCOMPLETE BLOCK DESIGNS WITH TWO-WAY CLASSIFICATION OF TREATMENTS

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1. Introduction. After Shah's [6] generalisation of the partially balanced incomplete block (PBIB) designs the author [5] gave a generalisation in a different direction. In this later generalisation, the association relation between treatments played a prominent part. This paper gives yet another genalisation of the PBIB designs. When the treatments to be administered are given, we may wish to classify them into groups with reference to each of the treatments, the classification being done on the basis of some characteristic of the treatments. Then we may insist that each of the treatments belonging to a particular group should occur together with the reference treatment of classification a particular number of times. From the point of view of easiness of analysis all such classifications of treatments are not desirable. A PBIB design insists on certain other conditions to be satisfied by these groups; for example, the constancy of the p_{jk}^i for any two treatments which are ith associates. We shall call the classification of treatments described above the classification according to association relation. After classifying treatments in this way, there still may exist some reason for classifying them into different groups independently of the former classification and then insisting that due consideration be given to this grouping in the lay-out of the design. In this paper, we shall consider one such type of PBIB design with two-way classification of treatments, in short written as PBIB (TW).

2. Notations and definition. Let α and β be two treatments. Let $\alpha \varepsilon G(a)$ denote "α belongs to group a" and $(\alpha, \beta) = l$ denote "α and β are lth associates". Then we may define our PBIB(TW) as follows:

DEFINITION. Given v treatments we group them into g groups containing $v_1, v_2, \cdots, v_g$ treatments such that the following conditions hold:
(1) Two treatments are either 1st, 2nd, $\cdots$, or mth associates;
(2) Each treatment belonging to group a has exactly $n(a, i)$ ith associates, $a = 1, 2, \cdots, g; i = 1, 2, \cdots, m$;
(3) Given two treatments, say α and β, $\alpha \varepsilon G(a)$, $\beta \varepsilon G(b)$, $(\alpha, \beta) = l$, the number of treatments common to the cth associates of α and dth associates of β in the order mentioned is given by $p(a, b; l; c, d)$ and is independent of the treatments α and β. If $\alpha \varepsilon G(a)$, $\beta \varepsilon G(b)$, $(\alpha, \beta) \neq l$, we define $p(a, b; l; c, d) = 0$. In general, $p(a, b; l; c, d) \neq p(a, b; l; d, c)$

Given such an association scheme we shall say that a PBIB(TW) of v treatments exists if the treatments can be arranged in b blocks of size k $(k < v)$ each such that

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