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ON MINIMUM VARIANCE UNBIASED ESTIMATION OF RELIABILITY

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1. Introduction. The purpose of the present paper is to present a simple method of deriving the unique minimum variance unbiased estimate (MVUE) of the reliability function associated with a life distribution. The widely applicable life distributions are the normal, one- and two-parameter exponential, gamma and Weibull distributions. A good deal of work has been done on this problem. Barton [1] estimated the probability that a normal variable will take a value between two points X_1 and X_2 in its range. Pugh [7] and Laurent [6] obtained the MVUE of reliability under the one-parameter and two-parameter exponential life distributions respectively. Tate [8] considered the two-parameter exponential, gamma and Weibull distributions and obtained MVUE's of some functions of the parameters. Recently, Basu [2] put forward a method of deriving MVUE of reliability and verified these estimates. The methods used by most of these authors consist of finding conditional distribution of a component of sample-observations given the sufficient statistics. In our present method, we find a statistic which is stochastically independent of the complete sufficient statistics and whose distribution can be very easily obtained. The MVUE is based on this distribution.

2. A general theorem for finding the MVUE. Let $\mathbf{X} = (X_1, X_2, \cdots, X_n)$ be a random sample of size n from a distribution function $F(x; \boldsymbol{\theta})$, where $\boldsymbol{\theta}$ denotes a vector of parameters. Let $\hat{\boldsymbol{\theta}}$ be a complete sufficient statistic for $\boldsymbol{\theta}$. Let $\psi(t, \boldsymbol{\theta})$ be a parametric function, t being a real number, and let

$$U(\mathbf{X}) = \lambda \quad \text{if } Z(\mathbf{X}) \geqq t, \qquad = 0 \quad \text{otherwise,} \tag{2.1}$$

for $Z(\mathbf{X})$ a function of $\mathbf{X}$ and λ any real number, be an unbiased estimate of $\psi(t, \boldsymbol{\theta})$. We introduce the following:

THEOREM 1. *If there exists a function $V(Z, \hat{\boldsymbol{\theta}})$ such that*
(i) *it is stochastically independent of $\hat{\boldsymbol{\theta}}$,*
(ii) *it is a strictly increasing function of Z for fixed $\hat{\boldsymbol{\theta}}$, and*
(iii) *its distribution function $H(x)$ is such that*

$$H(x) = 0 \quad \textit{for } x < a, \qquad = 1 \quad \textit{for } x > b.$$

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