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PROPERTIES OF POWER FUNCTIONS OF SOME TESTS CONCERNING DISPERSION MATRICES OF MULTIVARIATE NORMAL DISTRIBUTIONS[1]

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1. Introduction. The following testing problems relating the parameters of a p-variate normal distribution $N_p(\mu, \Sigma)$ are considered.

(a) Hypothesis: $\Sigma = \Sigma_0$, a known positive definite matrix.
Alternative: $\Sigma \neq \Sigma_0$.

(b) Hypothesis: $\Sigma = \sigma^2 I_p$, where σ^2 is unknown and I_p stands for the $p \times p$ identity matrix.
Alternative: $\Sigma \neq \sigma^2 I_p$.

(c) Hypothesis: $\Sigma = \Sigma_0$, a known pd matrix, $\mu = \mu_0$, a known vector.
Alternative: $\Sigma \neq \Sigma_0$ and/or $\mu \neq \mu_0$.

It has been shown that the likelihood ratio test is unbiased for the problems (b) and (c) but not for the problem (a); however, the "modified" likelihood ratio test for the problem (a) is not only unbiased but its power function has the usual monotonicity property. The above result for the problem (b) was first found by Gleser [4] using a different method; Cohen [2] obtained similar results for a more general version of (b).

It is further shown that the likelihood ratio test for testing the equality of the covariance matrices of two p-variate normal distributions is biased when the sample sizes are unequal.

All the above results are known to be true when $p = 1$. For simplicity, only the canonical forms of the above problems are considered.

2. Test of the hypothesis $\Sigma = I_p$. The following lemmas will be used to prove the main result. These lemmas are stated without proofs (which are fairly easy).

LEMMA 2.1. *For $S > 0$, $r > 0$, the region*

$$S^r \operatorname{etr}(-S/2) \geqq k$$

is equivalent to the region $s_1 \leqq S \leqq s_2$ where

$$s_1^r \operatorname{etr}(-s_1/2) = s_2^r \operatorname{etr}(-s_2/2) = k.$$

LEMMA 2.2. *Let S be a random variable such that the distribution of S/σ^2 is χ^2 with m degrees of freedom. For $r > 0$, let*

$$\beta(\sigma^2) = P[S^r \operatorname{etr}(-S/2) \geqq k; \sigma^2].$$

Then

$$d\beta(\sigma^2)/d(\sigma^2) \gtreqless 0$$

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