*The Annals of Mathematical Statistics*
1969, Vol. 40, No. 2, 356–365

# ON THE ADMISSIBILITY OF A RANDOMIZED SYMMETRICAL DESIGN FOR THE PROBLEM OF A ONE WAY CLASSIFICATION[1]

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**1. Introduction.** The paper by Kiefer [2], this paper, and Farrell [1], have resulted from a desire to begin giving a theoretical background to the choice of experimental designs in the analysis of variance. Thus it has been our purpose to formulate a decision theory meaning of admissibility for randomized designs and to use this definition to evaluate various procedures.

The present paper, as well as Kiefer [2], deals with the question of obtaining randomized designs having good power locally about the hypothesis. We show that in the case of the one way classification a certain randomized design, followed by use of the appropriate analysis of variance test, is an admissible procedure. The power function of this procedure locally about zero was investigated by Kiefer, op. cit., who gave definitions of optimality (to be distinguished from admissibility) and applied his definitions to the one-, two-, and three-way classifications. It is the author's conclusion, based on the contents of the papers cited, that a partial theory of design can be developed from convexity considerations provided one is willing to use randomized designs and base his choice upon the partial ordering of power functions.

The present paper has a main non-mathematical conclusion. This is, the practical statistician demands more than that his procedure have optimum power locally about zero. When Kiefer's paper, op. cit., was evaluated prior to publication he received criticism that no one would want to use such randomized procedures. The referee of the present paper writes "Perhaps it would help if some small effort was made . . . to give an example where one might conceivably want to perform an experiment by taking all observations from one class." Yet, in spite of these objections the mathematics is quite clear (but hard), that locally, good power is obtained using randomized designs as described.

We begin with a formulation of the admissibility concept. We suppose throughout that $N$ observations are to be taken. If there are $I$ "factors" under consideration then a design consists in part of specification of a vector $n^T = (n_1, \cdots, n_I)$ of integers, the design vector, such that if $1 \leqq i \leqq I$, then $n_i \geqq 0$ and $n_i$ observations are taken on the $i$th factor. Therefore $N = n_1 + n_2 + \cdots + n_I$. To each design vector $n$ we may form the set $\mathcal{R}_n$ of all risk functions of tests which use the observations taken according to the design vector $n$. $\mathcal{R}_n$ is then a convex (usually compact) set. The number of possible non-randomized designs is then the number $J$ of partitions of $N$ into $I$ non-negative integer summands. A randomized design

Received 28 March 1968; revised 18 November 1968.

[1] This research was supported in part by the Office of Naval Research under Contract Nonr 401(50). Reproduction in whole or in part is permitted for any purpose of the United States Government.