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A NOTE ON CHERNOFF-SAVAGE THEOREMS[1]

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Let $X_1, \cdots, X_m$; $Y_1 \cdots, Y_n$ be independent random samples from continuous df's F and G respectively; and let F_m and G_n be the corresponding empirical df's. Let $N = m + n$ and $\lambda_N = m/N$. Set

$$T_N = m^{-1}\sum_{i=1}^{N} c_{Ni}^* Z_{Ni} \tag{1}$$

where $\{c_{Ni}^* : 1 \leqq i \leqq N, N \geqq 1\}$ is a set of given constants and Z_{Ni} equals 1 (or 0) if the ith largest from $\{X_1, \cdots, X_m, Y_1, \cdots, Y_n\}$ is an X (or a Y).

The asymptotic normality of the class of statistics of the form (1) was studied first by Chernoff and Savage [1]. Since then several other approaches to this problem have been considered. In one of these approaches, [2], the authors presented some results (cf. Proposition 5.1, Corollary 5.1 and the related discussion in [2]) to indicate in what sense the results of [1] follow from those of [2]. The purpose of this note is to strengthen greatly these results by showing that a different decomposition of T_N makes Theorem 5.1 (a) of [2] more directly applicable and enables condition (i) of Theorem 5.1 (b) of [2] to be replaced by more easily verifiable conditions.

All notations undefined below are to be given their meaning according to Pyke and Shorack [2]. We recall only the following. For $N \geqq 1$ the L_N-process on [0, 1] is given by $L_N(t) = N^{\frac{1}{2}}[F_m \circ H_N^{-1}(t) - F \circ H^{-1}(t)]$ with $H_N = \lambda_N F_m + (1 - \lambda_n)G_n$ and $H = \lambda_N F + (1 - \lambda_N)G$; and the L_0-process is the natural limit of these processes. The signed measure ν and the related right continuous function J, which is of bounded variation on $[\epsilon, 1 - \epsilon]$ for all $\epsilon > 0$, satisfy

$$-\nu((a, b]) = J(b) - J(a) \qquad \text{for all} \quad 0 < a < b < 1.$$

Finally $\mathbf{Q}$ is the class of functions defined in [2]; an example of $q \in \mathbf{Q}$ is $q(t) = [t(1 - t)]^{\frac{1}{2}-\delta}$ for $\delta > 0$. Let

$$\tilde{T}_N = N^{\frac{1}{2}}[T_N - \mu] \tag{2}$$

where

$$\mu = \int_0^1 J d(F \circ H^{-1}). \tag{3}$$

Then

$$\tilde{T}_N = \tilde{S}_N + \theta_N + \epsilon_N \tag{4}$$

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