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ADMISSIBILITY OF THE USUAL CONFIDENCE SETS FOR THE MEAN OF A UNIVARIATE OR BIVARIATE NORMAL POPULATION

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1. Introduction. Let X be an m-dimensional vector distributed normally with mean vector θ and covariance matrix equal to the $m \times m$ identity matrix. A non-randomized confidence procedure C is a procedure, which assigns to each possible point x, a Lebesgue measurable subset $C(x, \cdot)$ of the parameter space within which θ is estimated to lie. Let $vC(x, \cdot)$ denote the Lebesgue measure of the set $C(x, \cdot)$. The usual procedure C_0 is a procedure in which the confidence sets $C_0(x, \cdot)$ are spheres of fixed volume, centered at the observed sample mean. C_0 has the property that amongst the class of confidence procedures with lower confidence level $(1 - \alpha)$, C_0 minimizes the maximum expected measure of the confidence sets viz.

$$\sup_\theta E_\theta vC(x, \cdot). \tag{1}$$

Stein (1962) raised the question whether the usual procedure is unique in having this property and conjectured that it is probably unique for $m = 1$, probably not so for $m \geqq 3$, the case $m = 2$ being doubtful. For the case $m \geqq 3$, the conjecture has already been shown to be true in a previous paper (Joshi (1967)). In this paper we now investigate the remaining cases $m = 1$ and $m = 2$.

A connected question is that of the admissibility of the usual procedure. Using the definition of admissibility of confidence sets formulated by Godambe (1961) and subsequently slightly revised by the author (1966) it is here shown that if apart from measurability there is no restriction on the form of the confidence sets, then no unique minimax or even admissible procedure can exist, as given any procedure another one uniformly superior to it can always be constructed. All the procedures so constructed however form a class called equivalence class such that for any two procedures in the class, for almost all x, the confidence sets differ from each other at most by null subsets of the parameter space. Admissibility or uniqueness of the minimax property can thus only pertain to the equivalence class which contains the usual procedure. Alternatively a unique minimax or admissible procedure can exist in the restricted class of confidence procedures for which the confidence sets are all convex sets or all open sets.

In the following remarks, therefore, the uniqueness or admissibility of the usual confidence procedure means the uniqueness or admissibility of the equivalence class which contains the usual procedure or alternatively its uniqueness

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