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ON FIXED PRECISION ESTIMATION IN TIME SERIES[1]

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0. Introduction. To the best of the authors' knowledge almost all of the work that has been done until the present on confidence intervals and confidence sets of fixed precision has either concerned independent (usually identically distributed) observations, or has been asymptotic in character.

In this paper we treat the problem of fixed length confidence intervals for the parameters of a discrete m-dependent stationary Gaussian process. Our main result is somewhat depressing; namely, that if m is unknown (i.e., the possible distributions consist of all m-dependent such processes for all m) such estimation is impossible. In fact it is impossible in a rather small subclass of these processes.

In this area there are, however, quite a few surprises. For example, the authors had conjectured that the main difficulty would arise in attempting to distinguish a case of independent observations with large mean and small variance from the case of 0 mean highly correlated observations with large variance. In both cases one would see a large first observation followed by a number of observations close by, and it appeared difficult to arrive at a stopping rule in which one could distinguish these two cases.

Our intuition appeared to be justified when we were able to show (Theorem 1) that for one class in which independence-large mean-small variance and high dependence-large variance cases were both included, there is no J-stage scheme for fixed length confidence interval estimation of the mean whose last $J-1$ sample sizes are determined by differences of values observed in previous stages. Recall that in Stein's two sample scheme the second sample size is determined by the first stage sample variance

$$k^{-1}\sum_{i=1}^{k} (X_i - \bar{X}_k)^2 = k^{-1} \sum_{i=1}^{k} (k^{-1} \sum_{j=1}^{k} [X_i - X_j])^2$$

which is a function of differences. In the general case of a stationary Gaussian process the variance of the sample mean (the usual estimator of the mean) is a function of the variances and covariances. We would expect that here too the actual sample sizes "should" be functions of previously observed differences, since sample covariances are also determined by differences. However our intuition does not hold up here, for we show in Theorem 2 that there is a two-stage scheme (whose second sample size is determined not by differences of 1st sample size values alone) for this problem.

In the next section we show that this case is really the exception and that if the nature of the dependence is not sufficiently well known, we are defeated. Over all, the results of this paper imply that making strong statistical inference

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