The Annals of Mathematical Statistics
1970, Vol. 41, No. 4, 1381–1383

BOOK REVIEWS

Correspondence concerning reviews should be addressed to the Book Review Editor, Professor James F. Hannan, Department of Statistics, Michigan State University, East Lansing, Michigan 48823.

M. Iosifescu and R. Theodorescu, *Random Processes and Learning*. Springer–Verlag, New York, 1969. x + 304 pp. $17.00.

Review by M. Frank Norman

University of Pennsylvania

This monograph consists of three long chapters: 1. A study of random sequences via the dependence coefficient. 2. Random systems with complete connections. 3. Learning.

The dependence coefficient on which Chapter 1 is based is Ibragimov's:

$$\phi(\mathscr{K}_1, \mathscr{K}_2) = \sup_{B\in\mathscr{K}_2}(\operatorname{ess\,sup}_{\omega\in\Omega}|P(B\,|\,\mathscr{K}_1)(\omega) - P(B)|),$$

where $\mathscr{K}_1$ and $\mathscr{K}_2$ are sub σ-algebras of a probability space $(\Omega, \mathscr{K}, P)$. This coefficient is used to formulate generalizations of the independence assumptions of a wide variety of classical limit theorems. For example, let $f_1, f_2, \cdots$ be a strictly stationary sequence of real random variables, let $\mathscr{K}_\Lambda$ be the σ-algebra generated by $\{f_i\colon i\in\Lambda\}$, and let

$$\phi(n) = \sup_{r\geqq 1}\phi(\mathscr{K}_{[1,r]}, \mathscr{K}_{[r+n,\infty)}).$$

Chapter 1 includes extensions, due to Iosifescu, of the Berry–Esséen theorem and the law of the iterated logarithm, in which the classical independence assumption is replaced by $\sum_{n=1}^{\infty}\phi^{\frac{1}{2}}(n) < \infty$ (plus $\phi(1) < 1$ in the latter theorem).

A random system with complete connections (indexed by the nonnegative integers) is a system $\{(W, \mathscr{W}), (X, \mathscr{X}), \{u_n\}_{n\geqq 0}, \{^nP\}_{n\geqq 0}\}$, where $(W, \mathscr{W})$ and $(X, \mathscr{X})$ are measurable spaces, u_n is a measurable transformation from $W\times X$ into W, and ^nP is a transition probability function on $W\times\mathscr{X}$. Associated with such a system and a $w\in W$ are stochastic processes $\xi_1, \xi_2, \cdots$ and $\zeta_0, \zeta_1, \cdots$ in X and W, respectively, such that $\zeta_0 = w$,

$$P(\xi_{n+1}\in A\,|\,\xi_n, \cdots, \xi_1) = {}^nP(\zeta_n; A),$$

and $\zeta_{n+1} = u_n(\zeta_n; \xi_{n+1})$ with probability 1. The system is *homogeneous* if u_n and ^nP do not depend on n. The process ζ_n is Markovian and, in the homogeneous case, has stationary transition probabilities. The term "complete connections" apparently refers to the intricate stochastic interdependence of the variables ξ_n.

The learning models considered in Chapter 3 are special random systems with complete connections. In the context of learning theory, the "state" variable ζ_{n-1}