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DISCRIMINATION OF POISSON PROCESSES[1]

BY MARK BROWN

Cornell University

0. Introduction. In [3] Gikhman and Skorokhod obtained necessary and sufficient conditions for absolute continuity of multidimensional independent increment processes. By the Lévy–Itô decomposition, an independent increment process $\{X(t),\ 0 \leqq t \leqq T\}$ can be decomposed into two independent components, an independent increment Gaussian process, and a process determined by the jumps of $\{X(t),\ 0 \leqq t \leqq T\}$, the jumps forming a Poisson process on $[0, T] \times E^n$. Thus in solving their problem, the authors obtained necessary and sufficient conditions for absolute continuity of Poisson processes on $[0, T] \times E^n$, for which the expected number of jumps of norm $> \varepsilon$ is finite for all $\varepsilon > 0$.

In this paper we consider the problem of absolute continuity of Poisson processes with σ-finite mean measures over general measure spaces. Then Gikhman–Skorokhod conditions for absolute continuity generalize to our case, but the proof of sufficiency does not, and a different proof is presented. We also obtain conditions for singularity of Poisson processes and show that two Poisson processes with mutually absolutely continuous mean measures are either mutually absolutely continuous or singular.

1. Define a Poisson $(\mathscr{X}, \mathscr{C}, \mu)$ process where $(\mathscr{X}, \mathscr{C})$ is a measurable space and μ a measure over $(\mathscr{X}, \mathscr{C})$, to be a random nonnegative integer valued (including $+\infty$) set function N on $(\mathscr{X}, \mathscr{C})$ having the property that for any k and corresponding nonnegative integers $r_1, \cdots, r_k$ and nonoverlapping $\mathscr{C}$ sets $C_1, \cdots, C_k$:

$$\Pr(N(C_j) = r_j, \quad j = 1, \cdots, k) = \textstyle\prod_1^k p(\mu(C_j), r_j) \tag{1}$$

where

$$\begin{aligned} p(\lambda, \alpha) &= \frac{\lambda^\alpha e^{-\lambda}}{\alpha!}, && \lambda < \infty, \qquad \alpha < \infty \\ &= 1 && \lambda = \infty, \alpha = \infty \text{ or } \lambda = 0, \alpha = 0 \\ &= 0 && \text{elsewhere.} \end{aligned}$$

It is easy to prove the existence of a Poisson $(\mathscr{X}, \mathscr{C}, \mu)$ process for μ σ-finite (see [1] page 1939).

Each realization of a Poisson $(\mathscr{X}, \mathscr{C}, \mu)$ process with μ σ-finite is of the form $N(C, \omega) = \sum_{t_i' \subset C} N(t_i'(\omega), \omega)$ where $\{t_i', i = 1, 2, \cdots\}$ is a random countable, collection of chunks of $\mathscr{X}$ (a chunk of $\mathscr{X}$ is a set $C \in \mathscr{C}$ such that $C' \subset C$

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