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A GENERALIZED DOEBLIN RATIO LIMIT THEOREM[1]

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1. Introduction. Let us consider a discrete time parameter Markov Process $\{X_k, k \geqq 0\}$ with stationary transition probability functions defined on a general measurable state space $(X, \mathscr{B})$ where $\mathscr{B}$ is a separable (countably generated) Borel field of subsets of X containing single point sets. Furthermore, assume the Harris recurrence condition.

CONDITION (C). There exists a sigma-finite measure μ defined on X, with $\mu(X) > 0$ such that for every $S \in \mathscr{B}$ with $\mu(S) > 0$, we have that

$$P[X_k \in S \text{ infinitely often} \mid X_0 = x] = 1$$

for all $x \in X$. This then implies the existence of an invariant sigma-finite measure Π on $(X, \mathscr{B})$, unique up to a constant multiple. (Note that the Π-measure of X may be infinite.) We denote the m-step transition probability from $x \in X$ to $S \in \mathscr{B}$ by $P^{(m)}(x, S)$.

Jain [8] has considered the Doeblin Ratio of these transition probabilities, namely,

$$\frac{\sum_{k=0}^{m} P^{(k)}(x, A_1)}{\sum_{k=0}^{m} P^{(k)}(y, A_2)}$$

where $x \in X$, $y \in X$, $A_i \in \mathscr{B}$ for $i = 1, 2$, with the aforementioned conditions on $(X, \mathscr{B})$. Here Jain proved that this ratio tends to $\Pi(A_1)/\Pi(A_2)$ as $m \to \infty$ for all x, y not in a set $N(A_1, A_2)$ where $\Pi[N(A_1, A_2)] = 0$. Isaac [7] then proved that the dependence of N on A_1 and A_2 could be removed if $A_i \subset S$ for $i = 1, 2$ with $0 < \Pi(S) < \infty$.

Krengel [11] considered the special case where the space consists of one discrete ergodic class which is recurrent, but further generalized the form of the Doeblin Ratio, i.e.,

$$\frac{\sum_{k=1}^{m} \sum_{x \in X} p_x P^{(k)}(x, A_1)}{\sum_{k=1}^{m} \sum_{x \in X} q_x P^{(k)}(x, A_2)}$$

where $\{p_x\}$ and $\{q_x\}$ represent initial probability distributions on the space. He proved that this ratio converges to $\Pi(A_1)/\Pi(A_2)$ as $m \to \infty$ when $\Pi(A_1) < \infty$, $\Pi(A_2) < \infty$ and the initial distributions satisfy certain conditions.

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