The Annals of Mathematical Statistics
1971, Vol. 42, No. 4, 1425–1428

SHORT COMMUNICATIONS

A NOTE ON ADMISSIBLE SAMPLING DESIGNS FOR A FINITE POPULATION

BY V. M. JOSHI

Secretary, Maharashtra Government, Bombay

1. Preliminary. Let U be a finite population of units $u_1, u_2, \cdots, u_N$. A sample s means any non-empty subset of U. A sampling design P is determined by defining a probability P on the set S of all possible samples s, $P(s)$ denoting the probability of the sample s. With each unit u_i is associated a variate value x_i, $i = 1, 2, \cdots, N$. $\mathbf{x} = (x_1, x_2, \cdots, x_N)$ denotes a point in the N-space R_N. Then for estimating the population total

$$T(\mathbf{x}) = \textstyle\sum_{i=1}^{N} x_i \tag{1}$$

the Horvitz–Thompson estimate (H–T estimate for short) is given by

$$\bar{e}(s, \mathbf{x}) = \sum_{i \in s} \frac{x_i}{\pi_i} \tag{2}$$

For unbiased estimation of $T(\mathbf{x})$ to be possible, it is a necessary condition that $\pi_i > 0$, $i = 1, 2, \cdots, N$. Throughout the following we restrict ourselves to the class C of sampling designs, for which this condition is satisfied and admissibility of a sampling design P means admissibility within the class C.

The variance of the H–T estimate is given by

$$V(\bar{e}, \mathbf{x}) = \sum_{i=1}^{N} \frac{x_i^{\,2}}{\pi_i} + 2 \sum_{1 \leqq i < j \leqq N} \frac{\pi_{ij}}{\pi_i \pi_j} x_i x_j - T^2(\mathbf{x}). \tag{3}$$

In (2) and (3), π_i and π_{ij} are respectively the inclusion probabilities of the units u_i and the pair of units u_i, u_j, i.e.

$$\begin{aligned} \pi_i &= \textstyle\sum_{s \ni i} P(s), \\ \pi_{ij} &= \textstyle\sum_{s \ni i,j} P(s), \qquad\qquad i, j = 1, 2, \cdots, N. \end{aligned} \tag{4}$$

In (2), (3) and (4) we have written $i \in s$ for $u_i \in s$, and similarly for $s \ni i$ and $s \ni i, j$.

The expected sample size for a given sampling design P is given by

$$v = \textstyle\sum_{s \in s} P_s n(s), \tag{5}$$

where $n(s)$ denotes the size of the sample s, i.e. the number of units u_i which belong to s.

Let P' be another sampling design and for P' let $V'(\bar{e}, \mathbf{x})$ and s' be the variance of the H–T estimate and the expected sample size. Suppose the sampling cost is

Received November 4, 1969; revised November 17, 1970.

Institute of Mathematical Statistics is collaborating with JSTOR to digitize, preserve, and extend access to *The Annals of Mathematical Statistics.* www.jstor.org