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DISTRIBUTED LAG ESTIMATION WHEN THE PARAMETER SPACE IS EXPLICITLY INFINITE-DIMENSIONAL

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1. Introduction. This paper discusses a number of results which were developed for application to the model

$$y = x * b + \varepsilon, \tag{1}$$

where x and ε are independent covariance–stationary stochastic processes with zero mean, b is a square summable sequence of real numbers, and "$*$" denotes convolution.[1] We will consider only the case with discrete time parameter.[2]

Where b is known to lie in some finite-dimensional linear space of sequences, estimation of b in (1) from a sequence of observations on y with x known can be carried out by least squares or generalized least squares. Even where restriction of b to such a space can only be regarded as approximately accurate common practice is to proceed with estimation as if the model were finite–dimensional without explicit concern for the effects of approximation.

There is one trivial case in which it is obviously possible to obtain consistent estimates with consistent confidence statements in an infinite–dimensional parameter space for b. Suppose b is known to lie within

$$S = \bigcup_{j=1}^{\infty} A_j, \tag{1A}$$

where A_j is a finite-dimensional linear space containing its predecessors. Any reasonable metric on S[3] will induce a topology on A_j equivalent to Euclidean topology. A natural procedure, then, is to start with A_1, estimating b in (1) on the assumption that b in fact lies in A_1. When the estimated confidence region for b has been reduced in radius (in the relevant metric) to $\frac{1}{2}$, proceed to A_2. Continue in this manner, shifting to A_{j+1} in every case only when the confidence region estimated within A_j has shrunk to a maximum radius of 2^{-j}. Even though the estimates and confidence regions may be inaccurate to start with, they are bound eventually to become accurate when we reach a j large enough so that the true value of b (call it b_0) lies in A_j. The only question is whether we can be sure that the procedure will make j go to infinity with probability one. This condition is easily verified for, say, the case where x and ε both have spectral densities bounded away from zero and infinity and are ergodic.

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[1] I.e., $a*b(t) = \sum_{s=-\infty}^{\infty} a(s)b(t-s) = \sum_{s=-\infty}^{\infty} b(s)a(t-s)$.

[2] The effects of approximating a model of the form (1) but with continuous time parameter by a similar model with discrete time parameter have been considered by the author (1971). A discussion of the implications of this paper's approach and results for econometric practice appear in Sims (1969).

[3] Any which makes it a topological vector space.