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ON THE ASYMPTOTIC DISTRIBUTION OF THE SEQUENCES OF RANDOM VARIABLES WITH RANDOM INDICES

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1. Introduction. In the present paper we investigate the asymptotic distribution of the sequence of random variables $(Y_{Nr})_{1 \leqq r < \infty}$ where $(Y_n)_{1 \leqq n < \infty}$ is a sequence of random variables and $(N_r)_{1 \leqq r < \infty}$ is a sequence of positive integer-valued random variables, both defined on a probability space $\{\Omega, \mathscr{K}, P\}$. About the behavior of the random indices we assume the following condition:

(C0) The sequence $(N_r/n_r)_{1 \leqq r < \infty}$ converges in probability to a positive random variable λ where $(n_r)_{1 \leqq r < \infty}$ is an increasing sequence of positive integer numbers tending to infinity when r tends to infinity.

The main problem is the following: *when does the sequence of random variables with random indices* $(Y_{Nr})_{1 \leqq r < \infty}$ *have the same asymptotic distribution as the sequence* $(Y_n)_{1 \leqq n < \infty}$? To simplify let us denote this main question by Q.

The first general result in this area (but with $\lambda \equiv 1$) was obtained by F. J. Anscombe (1952). Theorem 2 gives us the exact content of this result. Here condition (C4), known as "Anscombe's condition," is very important.

After the papers of W. Richter (1965a) (1965b), the latest general assertion in this direction (with λ arbitrary positive random variable) was formulated by J. Mogyoródi (1967). According to it an affirmative answer to the question Q is possible if both the classical Anscombe's condition (C4) and a mixing condition (similar to (C5)) hold. Unfortunately the proof given to this assertion is not correct. Professor Mogyoródi has kindly pointed out to me the error in his paper (1967). Therefore the validity of Mogyoródi's assertion is still an open question.

In the present paper we shall establish some theorems answering to the problem Q mentioned above. The main result is Theorem 3, similar to Mogyoródi's assertion. In fact, in Theorem 3 one condition is stronger while the other one is weaker than the analogous Mogyoródi conditions. At the same time, taking $\lambda \equiv 1$, Theorem 3 becomes just the classical Anscombe theorem.

Theorem 1 is more complicated but it is useful from the point of view of applications in the present topic. Simplifications are obtained if the random variables $(Y_n)_{1 \leqq n < \infty}$ are asymptotically independent with respect to λ (Theorem 4) or if the random variable λ takes on only discrete values (Theorem 6). Finally, all essential conditions are satisfied by sums of independent identically distributed random variables.

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