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LARGE SAMPLE TESTS FOR THE EQUALITY OF TWO COVARIANCE MATRICES[1]

By M. W. J. Layard

University of California, Davis

0. Introduction. It is well known that the standard normal-theory techniques for testing hypotheses about variances are extremely non-robust, even asymptotically, against departures from the assumed normality of the underlying distributions. As Box [4] points out, the reason for this frailty is that the relevant statistics, though asymptotically normal under general conditions, do not incorporate a corrective component to ensure the stability of the asymptotic variance under departures from normality. (By way of contrast, the t-statistic is "self-normalizing.")

Not surprisingly, the same difficulty arises in the multivariate case when we test hypotheses about covariance matrices. There is a well-developed body of normal-theory procedures for testing such hypotheses as the equality of two covariance matrices, independence of sets of variates, sphericity, etc., as well as for testing hypotheses about certain functions of covariance matrices, such as correlation coefficients and regression coefficients. It is the purpose of this paper to point out that the standard tests for equality of two covariance matrices are non-robust against departures from normality, and to discuss several procedures which are, at least asymptotically, robust. The non-robustness of normal-theory tests about correlation coefficients and the structure of covariance matrices will be discussed elsewhere.

In Section 1 we describe some notation and state some large-sample theory results which are needed in the sequel.

Section 2 demonstrates the non-robustness of two normal-theory tests for the equality of two covariance matrices. If S and T are the sample covariance matrices, then in the normal case the characteristic roots of ST^{-1} constitute a maximal invariant (under the full linear group), and the normal-theory tests which have been proposed employ functions of those roots. The two tests examined are the likelihood-ratio test, which uses a function of all the roots, and Roy and Gnanadesikan's test, which is based on the smallest and largest roots.

Section 3 considers four asymptotically robust procedures for testing the equality of two covariance matrices. The first of these, which is based on the elementary symmetric functions of the roots of ST^{-1} (where S and T are the

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