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A CONSISTENT ESTIMATION OF KERNEL FUNCTIONS IN THE MULTIPLE WIENER INTEGRALS

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1. Introduction. It is well known from the results of Itô [4] that any L^2-functional F of Brownian motion with $E[F] = 0$ has an orthogonal representation, i.e., $F = \sum_{p=1}^{\infty} I_p(f_p)$ where $I_p(f_p)$ is the pth degree multiple Wiener integral. The estimation of the kernel function f_p $(p = 1, 2, \cdots)$ is often required in solving various problems in nonlinear analysis and involves tremendous computations in the usual L^2-norm approximation. As a direct application of a result of Isaccson [3], we give here a consistent estimator for the kernel function f_p of L^2-functional of the form $F = \sum_{p=1}^{\infty} I_p(f_p)$.

2. Notations and preliminaries. Let $\{X(t, \omega)\}_{t \in [0,T]}$ be standard Brownian motion defined on a probability space $(\Omega, \mathscr{F}, P)$. Let B_t (for $t \in [0, T]$) denote the σ-field generated by sets of the form

$$E = \{\omega; [X(s_1, \omega), \cdots, X(s_n, \omega)] \in B^n\} \tag{2.1}$$

where $s_1, s_2, \cdots, s_n \in [0, t]$ and B^n is an n-dimensional Borel set. Let $\mathbf{B}_t$ denote the completion of B_t under P.

We shall write $L^2(\mathbf{B}_t)$ for $L^2(\Omega, \mathbf{B}_t, P)$, the Hilbert space of $\mathbf{B}_t$-measurable, real-valued functions square integrable with respect to P. We assume that $L^2(B_T)$ is separable. Let $L_t(X)$ denote the closed subspace of $L^2(\mathbf{B}_t)$ spanned by all finite linear combinations of the form $\sum_{i=1}^{n} c_i X(s_i, \omega)$ where the c_i's are real constants, and $s_1, s_2, \cdots, s_n \in [0, t]$.

We refer to Itô [4] for its definition and the various properties of the multiple Wiener integral;

$$I_p(f_p; t) = \int_0^t \cdots \int_0^t f_p(s_1, s_2, \cdots, s_p)\, dX(s_1)\, dX(s_2) \cdots dX(s_p) \tag{2.2}$$

for $f_p \in L^2([0, T]^p)$, where $L^2([0, T]^p)$ is the Hilbert space of Lebesgue square integrable functions on $[0, T]^p$. Denote $I_p(f_p) = I_p(f_p; T)$. The following results are due to Itô [4]. Any $F \in L^2(\mathbf{B}_T)$ can be expressed in the form:

$$F = \sum_{p=1}^{\infty} I_p(f_p) = \sum_{p=1}^{\infty} I_p(\tilde{f}_p)\,, \tag{2.3}$$

furthermore if

$$\sum_{p=1}^{\infty} I_p(f_p) = F = \sum_{p=1}^{\infty} I_p(g_p)\,, \tag{2.4}$$

then

$$\tilde{f}_p = \tilde{g}_p\,,$$

where $\tilde{f}_p(s_1, \cdots, s_p) = 1/(p!) \sum_{(\pi)} f_p(s_{\pi_1}, \cdots, s_{\pi_p})$, $(\pi) = (\pi_1, \cdots, \pi_p)$ running over all permutation of $(1, 2, \cdots, p)$.

When a sequence of random variables Y_n converges to a random variable Y in probability, we shall write $P \lim_{n\to\infty} Y_n = Y$.

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