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THE STOCHASTIC APPROXIMATION APPROACH TO A DISCRIMINATION PROBLEM

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1. Introduction. Let the function M map the real line R into R; let $\{Y(x), x \in R\}$ be a family of independent random variables, where $Y(x)$ has distribution function $F(\cdot \mid x)$ with

$$\int_{-\infty}^{\infty} y\, dF(y \mid x) = M(x) \qquad \text{and} \qquad \int_{-\infty}^{\infty} [y - M(x)]^2\, dF(y \mid x) = \sigma^2(x) < \infty ,$$

for every $x \in R$.

Let $\alpha \in R$ be a particular value and suppose that there exists a point $\theta \in R$ such that $[M(x) - \alpha](x - \theta) > 0$, for every $x \neq \theta$.

Let $\{Z_i, i = 1, 2, \cdots\}$ be a family of independent, identically distributed random variables, independent of $\{Y(x), x \in R\}$, with distribution function $G(\cdot)$, where $\int_{-\infty}^{\infty} z\, dG(z) = \alpha$ and $\int_{-\infty}^{\infty} (z - \alpha)^2\, dG(z) = \gamma^2 < \infty$.

Suppose that the regression function M and the values θ and α are unknown and that after observing some Z's and $Y(x)$'s at various x values, one wishes to estimate θ.

For ease in presentation, observations corresponding to the Z's will be called "control observations" and observations corresponding to the $Y(x)$'s will be called "test observations."

The discrimination problem described above would be fairly straightforward if the form of the regression function M and the distribution functions $F(\cdot \mid x)$, $x \in R$, and G were known. For example, if M is linear, G is normal and $F(\cdot \mid x)$ is normal with $\sigma^2(x) = \sigma^2$, for every $x \in R$, then a solution to the discrimination problem is given by the well-known Fieller estimate (Fieller [4]).

It is important from a practical point of view, however, to look for a solution to the discrimination problem when little is known a priori about the functions M, G, and F.

For example, suppose a scientist is comparing two drugs, a test drug and a control drug, and that he is interested in designing a biological assay to estimate the number of dose units of the test drug necessary to elicit the same mean response as the standard dose of the control drug. Suppose, further, that the experimenter knows little about the shape of the response function associated with the test drug and about the probability distribution of responses at any one dose level of either drug.

Make the following notational identifications. Let an observed response to the control drug administered at the standard dose level correspond to the random variable Z with mean α. Let the observed response to the test drug at dose level x correspond to $Y(x)$ with mean $M(x)$. Let θ be the dose level of the test drug

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