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BAYESIAN INFERENCE IN LINEAR RELATIONS

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1. Introduction. The statistical analysis of a linear relation among several unobserved variables, when the observations are all subjected to error, has a long history, going back to a paper by R. J. Adcock published in 1877. The early writers on this subject, notably R. J. Adcock (1878), C. H. Kummel (1879) Karl Pearson (1901) and M. J. van Uven (1930) were mainly concerned with the derivation of least squares estimators. Modern statistical methods were used for the first time in the analysis of linear relations by A. Wald in paper published in 1940. As was pointed out by J. Neyman in 1937, if no replications are available and the errors and the unobserved variables are independent and have Gaussian distributions, then the linear relation may be unidentifiable in the sense that even a complete knowledge of the sampling distributions of the observed random variables is not sufficient to determine the linear relation, because the number of parameters in the linear relation model is, in such cases, greater than the number of parameters which determine the sampling distributions. If we try to solve this problem by Bayesian methods, we cannot expect that the posterior distribution will be consistent in the sense of converging to the true values of the parameters when the sample size increases indefinitely. However, the assumption that the unobserved variables are independent is unrealistic in many cases, particularly in time series analysis, and better alternative models are needed. Of course, under certain additional conditions there are no problems of identifiability. Thus, J. Kiefer and J. Wolfowitz (1956) showed that, under certain conditions of identifiability, when no replications are available and the unobserved variables have probability distributions, the method of maximum likelihood, if properly applied, will yield not only consistent estimators of the linear relation, but, what is even more remarkable, it will yield also consistent estimators of the probability distribution of the unobserved random variables. However, Kiefer and Wolfowitz do not give explicit expressions for the maximum likelihood estimators.

In experimental work it is usually possible to replicate the observations. Replicated experiments can be analyzed without great difficulties, because we can easily obtain from them estimators of the experimental errors which have known distributions. The case in which replicated observations are available was considered systematically for the first time by G. W. Housner and J. F. Brennan (1948), but the first important work on this case was done by John W.

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