

## CORRECTION

### BROWNIAN MODELS OF OPEN PROCESSING NETWORKS: CANONICAL REPRESENTATION OF WORKLOAD

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The paper above contains a substantial error, as shown by example in Section 7.1 of [2]. To correct this error one needs to add a restriction on the data of the model being analyzed, thus making explicit an assumption that is implicit in the original development. To enable a precise statement of that additional assumption, the next paragraph reviews some of the paper's notation and set-up.

Let  $r$ ,  $m$  and  $n$  be positive integers,  $R$  an  $m \times n$  input-output matrix,  $A$  a nonnegative  $r \times n$  capacity consumption matrix, and  $\lambda$  an  $m$ -vector of exogenous input rates. All of the paper's formal propositions relate, at least implicitly, to the following linear program, called the *static planning problem* (here displays are numbered exactly as in the original paper, for ease of reference): choose a scalar  $\rho$  and an  $n$ -dimensional column vector  $x$  of average activity rates so as to

$$(2.1) \quad \text{minimize } \rho$$

$$(2.2) \quad \text{subject to } Rx = \lambda, \quad Ax \leq \rho e \quad \text{and} \quad x \geq 0,$$

where  $e$  is the  $m$ -vector of ones. The dual of (2.1)–(2.2), or to be more precise, one formulation of its dual, is as follows: choose an  $m$ -dimensional row vector  $\mu$  and  $r$ -dimensional row vector  $\pi$  so as to

$$(2.3) \quad \text{maximize } \mu\lambda$$

$$(2.4) \quad \text{subject to } \mu R \leq \pi A, \quad \pi e = 1 \quad \text{and} \quad \pi \geq 0.$$

In the primal problem (2.1)–(2.2) one interprets  $\lambda_i$  ( $i = 1, \dots, m$ ) as the average rate at which a system manager is obliged to consume input  $i$ , or the average rate at which type  $i$  input must be processed. On page 80 of the paper under discussion, the vector  $\lambda$  is assumed to be nonnegative and nontrivial, but the nonnegativity is never actually used in the ensuing development; if  $\lambda_i$  were negative one could interpret its absolute value as the average rate at which the system manager is obliged to *supply* material  $i$ .

To articulate the additional assumption referred to earlier, let us define the polyhedral cone

$$V = \{v \in \mathbb{R}^m : Rx = v \text{ for some } x \geq 0\}.$$