The Annals of Probability
2020, Vol. 48, No. 3, 1592–1595
https://doi.org/10.1214/19-AOP1394
Main article: https://doi.org/10.1214/009117904000000207

FLOWS, COALESCENCE AND NOISE. A CORRECTION

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Dr Georgii Riabov communicated to us a nonuniqueness example (see his paper [3]) of a random map which contradicts Remark 1.7 in our paper [1]: Flows, coalescence and noise. Ann. Probab. 32 (2004), no. 2, 1247–1315.

In short, with the notation of our paper, the counterexample is as follows.

COUNTEREXAMPLE TO REMARK 1.7 IN [1]. Let φ be a random variable in F (i.e., φ is a random measurable mapping on a compact metric space M) of law Q such that $M \times \Omega \ni (x,\omega) \mapsto \varphi(x,\omega) \in M$ is measurable. Suppose that Q is regular and let $\mathcal{J}$ be a regular presentation of Q. Let X be a random variable in M independent of φ. Out of φ and X, define $\psi \in F$ by $\psi(x) = \varphi(x)$ is $x \neq X$ and $\psi(x) = X$ is $x = X$. Then $M \times \Omega \ni (x,\omega) \mapsto \psi(x,\omega) \in M$ is measurable. Suppose also that the law of X has no atoms, then (reminding the definition of $\mathcal{F}$) ψ and X are independent and the law of ψ is Q. Note that $\psi(X) = X$ and (except for very special cases) we will not have that a.s. $\mathcal{J}(\psi)(X) = \psi(X) = X$.

This leads us to propose a correction to the first two sections of the paper. *In order to preserve the one-to-one correspondence between laws of stochastic flows of maps, or kernels and consistent systems of Feller semigroups, keeping the same set of notation, we slightly reinforce the definition of a stochastic flow.*

For flows of mappings the new definition is as follows.

DEFINITION 1. Let $(\Omega, \mathcal{A}, \mathsf{P})$ be a probability space and let $\varphi = (\varphi_{s,t}, s \le t)$ be a family of $(F, \mathcal{F})$-valued random variables such that for all $x \in M$ and all $t \in \mathbb{R}$, P-a.s. $\varphi_{t,t}(x) = x$. For $t \ge 0$, denote by Q_t the law of $\varphi_{0,t}$. The family φ is called a stochastic flow of mappings if for all $t \ge 0$, Q_t is regular and if the following properties are satisfied by φ:

(a) For all $s \le u \le t$, all $x \in M$ and all measurable presentation $\mathcal{J}_{t-u}$ of Q_{t-u}, P-almost surely, $\varphi_{s,t}(x) = \mathcal{J}_{t-u}(\varphi_{u,t}) \circ \varphi_{s,u}(x)$. (Cocycle property)
(b) For all $s \le t$, the law of $\varphi_{s,t}$ is Q_{t-s}. (Stationarity)
(c) The flow has independent increments, that is, for all $t_1 \le t_2 \le \cdots \le t_n$, the family $\{\varphi_{t_i,t_{i+1}}, 1 \le i \le n-1\}$ is independent.
(d) For all $f \in C(M)$ and all $s \le t$,

$$\lim_{(u,v)\to(s,t)} \sup_{x\in M} \mathsf{E}[(f \circ \varphi_{s,t}(x) - f \circ \varphi_{u,v}(x))^2] = 0.$$

(e) For all $f \in C(M)$ and all $s \le t$,

$$\lim_{d(x,y)\to 0} \mathsf{E}[(f \circ \varphi_{s,t}(x) - f \circ \varphi_{s,t}(y))^2] = 0.$$

Received July 2019.