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## REJOINDER: "FIBER DIRECTION ESTIMATION, SMOOTHING AND TRACKING IN DIFFUSION MRI"

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The discussants have raised many insightful comments, and we shall respond to them in the alphabetical order of their last names. Briefly, these comments can be grouped into three major themes: clarification of the details and limitations of our techniques (mostly from Kang and Li), discussion of possible directions for future work (mostly from Lazor) and connection to the Watson mixture distributions (mostly from Schwartzman).

JIAN KANG AND LEXIN LI: We thank Jian and Lexin for their detailed and constructive comments. Due to space constraints, we only respond to some of their comments below. Some of these responses help further clarify the details of our techniques.

Label switching: Jian and Lexin raise a good question about identifiability under label switching. In a more precise description, the proposed parametrization is only identified up to a label switching. However, we feel that it is unnecessary to further identify the labels in voxel-wise estimation. In our view, there are two types of labeling that one can assign to the directions. One is constructed to solely ensure the absolute identifiability. But such further identification of direction labels does not carry any practical meaning, and has little relevance in the estimation and subsequent steps. The second type of labels has physical meaning, here the fiber membership. But this labeling cannot be ascertained based on measurements within a single voxel. In our procedure, they are identified through the clustering procedure in the smoothing step.

Estimation of  $S_0$  and  $\sigma$ : We greatly appreciate the effort spent on the discussion and numerical experiments. We agree that appropriate estimation of  $S_0(\cdot)$  and  $\sigma$  using both b0 and non-b0 images will likely improve their estimation quality. This seems to be suggested by the numerical finding in Jian and Lexin's discussion. But we do not fully understand their numerical results due to lack of details such as the exact definition of the mean square errors of the direction estimate. Therefore, we confine our discussion to why  $S_0(\cdot)$  and  $\sigma$  are estimated separately in our paper. The major reason is computational simplicity and efficiency. As discussed in our response to Armin, the voxel-wise estimation is the most computational expensive