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DISCUSSION OF "FREQUENTIST COVERAGE OF ADAPTIVE NONPARAMETRIC BAYESIAN CREDIBLE SETS"

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First I would like to congratulate the authors Botond Szabó, Aad van der Vaart and Harry van Zanten for a fine piece of work on the extremely important topic of frequentist coverage of adaptive nonparametric credible sets. Credible sets are used by Bayesians to quantify uncertainty of estimation, which is typically viewed as more informative than point estimation. Such sets are often easily constructed, for instance, by sampling from the posterior, while confidence sets in the frequentist setting may need evaluating limiting distributions, or resampling, which needs additional justification. Bayesian uncertainty quantification in parametric problems from the frequentist view is justified through the Bernstein–von Mises theorem. In recent years, such results have also been obtained for the parametric part in certain semiparametric models, guaranteeing coverage of Bayesian credible sets for it. However, as mentioned by the authors, inadequate coverage of nonparametric credible sets has been observed [Cox (1993), Freedman (1999)] in the white noise model, arguably the simplest nonparametric model. A clearer picture emerged after the work of Knapik, van der Vaart and van Zanten (2011) that undersmoothing priors can resolve the issue of coverage; see also Leahu (2011) and Castillo and Nickl (2013).

In the present paper, the authors address the issue of coverage of credible sets in a white noise model under the inverse problem setting, when the underlying smoothness (i.e., regularity) of the true parameter is not known, so a procedure must adapt to the smoothness. The authors follow an empirical Bayes approach where a key regularity parameter in the prior is estimated from its marginal likelihood function. As the authors mentioned, undersmoothing leads to inferior point estimation and is also difficult to implement when the smoothness of the parameter is not known. We shall see that the issue of coverage can also be addressed by two other alternative approaches.

Before entering a discussion on the contents of the paper, let us take another look at the coverage problem for Bayesian credible sets in an abstract setting. Suppose that we have a family of experiments based on observations $Y^{(n)}$ and indexed by a parameter $\theta \in \Theta$, some appropriate metric space. Let ϵ_n be the minimax convergence rate for estimating θ. Let $\gamma_n \in [0, 1]$ be a sequence which can be fixed or may tend to 0. For some $m_n \to \infty$, typically a slowly varying sequence, the goal is to find a subset $\mathcal{C}(Y^{(n)}) \subseteq \Theta$ such that uniformly on $\theta_0 \in \mathcal{B}$:

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