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Discussion of "Statistical Modeling of Spatial Extremes" by A. C. Davison, S. A. Padoan and M. Ribatet

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We congratulate the authors for producing such a helpful and comprehensive overview paper of a rapidly developing and important area. The starting point for inference in spatial extreme value problems is to identify which features of the process require modeling. In certain applications, for example, in the generation of return period maps, only the marginal behavior is of concern. For such applications, the covariate hierarchical/latent variable models reviewed in Section 4 are ideal. However, if there is any dependence in the process between sites, then care needs to be taken when assessing the uncertainty in the estimated marginal distribution; the composite likelihood procedures detailed in Section 6.2 can also be exploited in this context when one wishes to avoid assumptions on the form of the spatial dependence. As the authors point out, however, if interest lies in modeling the joint occurrence of extremes over a region, then the dependence structure of the spatial process needs to be explicitly modeled. The most widely used approach in such cases is to model the process as a max-stable process. Here we will explore the suitability of this framework for modeling spatial extremes, since these processes are quite restrictive in their assumptions. Specifically, all finite dimensional distributions of a max-stable process are multivariate extreme distributions. Even simply in the bivariate case, this corresponds to the variables being exactly independent or asymptotically dependent. Consequently, the broad class of asymptotically independent variables is precluded under the modeling assumptions of max-stable processes.

First consider diagnostic testing for the process being max-stable. From our experience we feel that in many applications, max-stable processes are assumed to be appropriate without testing their suitability for the data. A partial justification for this is that often it is pointwise maxima that are being modeled, and so just as one appeals to the marginal limit theory to justify fitting the GEV distribution site-wise, it seems natural to appeal to the limit theory for the dependence structure also. However, we would typically not fit the GEV to the margins blindly, but look to assess its suitability through diagnostics such as Q-Q plots. To our knowledge, there currently are no diagnostics for testing if the process is max-stable, and so this discussion contribution aims to provide such a test.

Suppose that $\{Y(\mathbf{x}) : \mathbf{x} \in A\}$ is a max-stable process on the region A with standard Gumbel marginal distributions for all $\mathbf{x}$. This is simply attained through a pointwise log transformation of the more commonly-assumed standard Fréchet margins. As the process will typically only be observed at a finite set of sites $\mathbf{x}_1, \ldots, \mathbf{x}_m$, then all that can be tested for in practice is that the joint distribution of $\{Y(\mathbf{x}_j) : j \in \Delta\}$, where $\Delta = \{1, \ldots, m\}$, follows a multivariate extreme value distribution. Then for any $D \subseteq \Delta$ we have that the joint distribution function for $\{Y(\mathbf{x}_j) : j \in D\}$ is

$$G_D(\mathbf{y}) = \exp[-V_D\{\exp(\mathbf{y})\}],$$

where V_D is the associated exponent measure; see Section 2.3. A key property of G_D, due to max-stability, is that the distribution of $Y_D = \max_{j \in D} Y(\mathbf{x}_j)$ is

$$H_D(y) = G_D(y\mathbf{1}) = \exp[-\exp\{-(y - \mu_D)\}].$$

This is a Gumbel distribution with location parameter $\mu_D = \log V_D(\mathbf{1})$ where $0 \le \mu_D \le \log(|D|)$, due to bounds on the exponent measure. It follows that $Z_D = Y_D - \mu_D$ is standard Gumbel for all $D \subseteq \Delta$. The idea of the diagnostic is to pool values of Z_D over replicates of the max-stable process and over all $D \subseteq \Delta$, of a particular cardinality $k = |D|$, and test using a P-P plot whether the variables Z_D follow a standard Gumbel distribution. This enables an assessment of the kth dimensional properties of the process. Here we look at $k = 2, 3, 4, m$.

There are a few practical issues to address. As the value of μ_D is unknown for all D with $|D| \ge 2$, these

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