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Discussion of "Estimating Random Effects via Adjustment for Density Maximization" by C. Morris and R. Tang

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We thoroughly enjoyed reading this excellent authoritative paper full of interesting ideas, which should be useful in both Bayesian and non-Bayesian inferences. We first discuss the accuracy of the ADM approximation to a Bayesian solution in a real-life application and then discuss how some of the ideas presented in the paper could be useful in a non-Bayesian setting.

HOW DOES THE ADM WORK IN A REAL APPLICATION?

Although the main objective of this paper is to make inferences on the high-dimensional parameters or the random effects θ_i, the authors note that the success of the Bayesian method lies on the accurate estimation of the shrinkage parameters B_i since they appear linearly in the expressions for the posterior mean and posterior variance of θ_i when the hyperparameters are known. Thus, we assess the accuracy of the ADM approximation, given in Section 2.8, relative to the standard first-order Laplace approximation, in approximating the posterior distribution of the shrinkage factors for the hierarchical model (1)–(2). This model, commonly referred to as the Fay–Herriot model in the small area literature, was used by Fay and Herriot (1979) in order to combine survey data and different administrative records in producing empirical Bayes estimates of per-capita income of small places. Since then the Fay–Herriot model and its variants have been used in various federal programs such as the Small Area Income and Poverty Estimates (SAIPE) and the Small Area Health Insurance estimates (SAHIE) programs of the U.S. Census Bureau.

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For purposes of evaluation, we consider the problem of estimating the proportion of 5- to 17-year-old (related) children in poverty for the fifty states and the District of Columbia using the same data set considered by Bell (1999). We choose two years (1993 and 1997) of state-level data from the SAIPE program. In 1993, the REML estimate of A is positive while in year 1997 it is zero. The choice of these two years will thus give us an opportunity to assess the accuracy of the ADM approximations in two different scenarios.

We assume the standard SAIPE state-level model in which survey-weighted estimates of the proportions follow the two-level model given by (1)–(2). The survey-weighted proportions are obtained using the Current Population Survey (CPS) data with their variances V_i estimated by a Generalized Variance Function (GVF) method, following Otto and Bell (1995), but assumed to be known throughout the estimation procedure. We use the same state-level auxiliary variables x_i (a vector of length 5, i.e., $r = 5$), obtained from Internal Revenue Service (IRS) data, food stamp data and Census data that the SAIPE program used for the problem. We assume the uniform prior on $\boldsymbol{\beta}$ and superharmonic (uniform) prior on A, as used in the Morris–Tang paper.

For the presentation of our results, we consider a selection of four states—California (CA), North Carolina (NC), Indiana (IN) and Mississippi (MS)—considered by Bell (1999). This selection represents both small (i.e., large V_i) and large (i.e., small V_i) states and thus should give us a fairly general idea of the degree of accuracy of the Laplace and ADM approximations with varying V_i when compared to the exact posterior distributions of the shrinkage factors obtained by one-dimensional numerical integration.

First, consider the year 1993 when the REML estimate of A is positive (1.7). The exact posterior distributions of the shrinkage factors, ADM approximations and the first-order Laplace approximations (Kass and Steffey, 1989) are plotted in Figure 1. The solid curves in Figures 1 and 2 are the exact posterior distributions