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Discussion of "Objective Priors: An Introduction for Frequentists" by M. Ghosh

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1. INTRODUCTION

Professor Ghosh has produced a very useful, interesting piece of work which (i) argues that Bayesian results with objective priors may be interesting for frequentist statisticians, (ii) reviews two useful (unrelated) techniques which find application in the derivation of objective priors, (iii) introduces a family of divergence priors which is claimed to include reference priors, (iv) reviews matching priors, and (v) demonstrates that these ideas may produce new objective priors. I will comment in turn on each of these points.

2. OBJECTIVE BAYESIAN STATISTICS

Professor Ghosh states that "with enough historical data, it is possible to elicit a prior distribution fairly accurately." I believe this is a (possibly misleading) overstatement, an example of wishful thinking. In practice, useful prior elicitation is limited to small text-book models with very few parameters. I have never seen a proper elicitation job in moderately complex conventional models (say a logistic regression), let alone in really complex problems. In optimal circumstances, one may be able to elicit a proper joint prior for a couple of parameters of interest, but one is then forced to assume some form of objective conditional prior for the many nuisance parameters typically present in any real application. Some people then use a "flat" prior, typically a limiting form of some conjugate family of priors; but this is a very dangerous procedure, for one does not control the implications of the choice made, and may result in severely biased, or even improper posteriors. There is simply no substitute for the search of a well-motivated objective prior.

The author further states that "Bayesian methods, if judiciously used, can produce meaningful inferences based on... objective priors" and makes reference to several problems where frequentist methods fail to produce sensible answers, while objective Bayesian methods certainly succeed. I surely agree with this, but I find this to be an understatement. Ever since Wald (1950) proved that to be admissible (a frequentist concept!) a procedure *must* be Bayesian, people have found, over and over again, that (as could have been expected from this general result) the frequentist performance of objective Bayesian procedures is typically very good, and often better than that of the procedures derived from *ad hoc* frequentist methods. Actually, one could well invert the conventional teaching of mathematical statistics, by teaching first objective Bayesian methods (motivated from first principles), and then introducing frequentist ideas and proving that, under replication, objective Bayesian methods *also* perform very well.

3. ASYMPTOTIC EXPANSIONS AND SHRINKAGE

Theorem 1 is a very useful result... when it is applicable. This essentially requires conditions for the posterior to be asymptotically normal, and we all know many important examples where this is *not* the case. It is conceivable that alternative asymptotic expansion may similarly be obtained in those "nonregular" cases, and I would like Professor Ghosh to comment on this.

The shrinkage argument introduced by J. K. Ghosh was a welcome addition to the mathematical statistician toolkit. It often provides an elegant, efficient procedure to obtain conditional expectations. This is another example of the power of techniques based on working on sequences of priors based on compact sets, a procedure pioneered in the construction of reference priors, and developed in detail in Berger, Bernardo and Sun (2009), where these types of sequences are used to derive reference priors in completely general situations, with no assumptions of asymptotic normality.

4. DIVERGENCE PRIORS

Professor Ghosh recalls that in the original paper on reference priors (Bernardo, 1979), these are ob-

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