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Comment on Article by Hoff

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I would like to start by congratulating Peter for sharing a stimulating paper on Bayesian multidimensional modeling of Gaussian data. The paper shows in a constructive and didactical manner that the array normal distribution shares most of the nice properties of the well-known multivariate normal and the matrix normal distributions. One of the nicest features of this new class of distributions is its flexibility to easily accommodate extensions for time series, spatio-temporal and other types of commonly known statistical models for multivariate and matrix-variate data. The following points are very subjective and relate to my own research interest, but I have no doubt other readers will find the paper very interesting and will formulate their own questions and make connections to their own research on multidimensional modeling of Gaussian data.

Seemingly unrelated regressions. One important class of of separable covariance structures in econometrics is the class of seemingly unrelated regression (SUR) models, popularized by Arnold Zellner in the 1960s (Zellner, 1962, 1963), which are now common place, for instance, in i) various versions of vector autoregressive (VAR) modeling of macroeconomic and financial econometrics data and in ii) multinomial probit modeling of microeconomic and marketing data.

Factor models. In Lopes, Salazar and Gamerman (2008) and Lopes *et al.* (2010) we handle space-time dependences and multi-level dependences, respectively, through the prior distributions on factor loadings and common factors themselves. Calder (2007), for instance, models multiple pollutant levels at multiple locations and multiple time periods. The discussion of the current paper suggests some basic factor model extensions to the array normal distribution inspired by Lathauwer *et al.*'s (2000) higher order singular value decompositions. Indeed, this is a very important extension and it would be of great value to search for a common ground between these highly "regularlized", dynamic and hierarchical factor models and the array factor normal models. For a more concrete example, consider the international trade of Section 4. A hierarchical model with a similar flavor to the ones in Lopes *et al.* (2010) would be

$$y_{ijt}|f_t \sim N(\beta_{ij} f_{ijt}, H) \quad \text{with} \quad \beta_{ij}|\theta_i \sim N(\theta_i, R)$$

for $i, j = 1, \ldots, 30$, $i \neq j$, $H = \text{diag}(h_1^2, \ldots, h_6^2)$ and, for simplicity, β_{ij} loading vectors of dimension 29. The matrix R would take into account the countries' spatial (possibly economical, not geographical) co-dependence, while θ_i would take into account country specific co-variates. The next level of the hierarchy would assume that $\theta_i \sim N(\theta_0, V_0)$, with θ_0 and V_0 taking standard multivariate normal and inverse-Wishart priors. Finally, the factor score f_{ijt} would follow, say, a standard first order autoregression, i.e. $f_{ijt} \sim N(\phi_{ij} f_{ij,t-1}, \tau_{ij}^2)$ and parameters (ϕ_{ij}, τ_{ij}^2) would follow standard univariate normal and

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