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Rejoinder: Likelihood Inference for Models with Unobservables Another View

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1. INTRODUCTION

First we should like to thank the editor for allowing us to respond to interesting discussions from the discussants, Molenberghs, Kenward and Verbeke (MKV), Louis and Meng, for the effort they have put into their replies, and for the many important points that they have raised.

We view statistics as comprising relationships between models and data, where a statistical model is a formal mathematical formula which in some sense represents the patterns in the data. It represents a tool underlying the process of "making sense of figures." There are two processes linking models and data. The first, which we term the forward process, can be written as

$$\text{model} \longrightarrow \text{data}.$$

This stands for, "given a model, what would the data that it generates look like?" We call this process statistical modelling and it forms the basis of probability theory. The second process, which we term the backward process, can be written as

$$\text{model} \longleftarrow \text{data}.$$

This stands for, "given data, and a (guessed) model what can we say about the parameters in that model?" We call this process statistical inference, and it is displayed in Efron's (1998) triangular diagram for 21st-century statistical research, involving the three schools, Fisherian, Bayesian and Frequentist. The process of inference involves two procedures, namely model fitting and model checking. In the first we find values for the parameters in the model that fit the data best, and in the second we use probability theory to check whether the fit and, therefore, the assumed model is acceptable, by looking at the distribution of a suitable badness-of-fit statistic. Model checking could lead to a new model involving the two processes.

Among the discussants, MKV seem to suggest that data contain information only about the parameters in the marginal likelihood, but not about the unobservables (random effects) in the h-likelihood. Louis and Meng say that extended likelihood such as h-likelihood does indeed carry information about the unobservables, but that nevertheless the Bayesian approach is best suited for such inferences. We hope to show how the ideas can be combined together in the h-likelihood framework to give a new type of statistical inference. We shall try to make clear the inferential status of our framework.

The Bayesian framework is a well-defined mathematical structure about which theorems can be proved. However, it requires a statement of subjective prior belief about the unknown parameters which we are unable to provide. Of course many attempts have been made to define "objective" priors, but we believe them not to have been successful. As Barnard (1995) used to stress, in scientific inference the aim is to look for objective conclusions that scientists can agree upon. Senn (2008) puts it more strongly when he writes, "In fact the gloomy conclusion to which I am drawn on reading de Finetti (1974) is that ultimately the Bayesian theory is destructive of any form of public statistics." An alternative description is that we are looking across data sets for significant sameness, structures that remain unchanged when external conditions vary, a view which has been strongly propagated by Ehrenberg (1975). Another problem of using priors on parameters is that however many data are collected, no information is added regarding the parameters in the prior. In contrast to the many possible priors in Bayesian framework, in our system there is only one corresponding prior likelihood (Pawitan, 2001) for parameters, namely $L(\theta) = 1$, and as data grow information is accumulated on all parameters in the model. Model checking is a vital part of inference and we regard accumulated information as necessary for model

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