## Introduction to the Special Issue: Bayes Then and Now

## **Christian P. Robert and George Casella**

The arrival of David Bellhouse's autobiography of Sir Thomas Bayes at the *Statistical Science* editorial office triggered an idea that eventually became this issue. Although there is a distinct difference in the exact interpretation of "Bayes" in the two sections (as one refers to the man and the other to the subject), we thought that editorial privilege was in order.

While there is some uncertainty about the date of his birth, Thomas Bayes was born approximately 300 years ago, and, although the connection with this English mathematician of the 18th century is not entirely clear (see Stigler, 1983), a field of Statistics has taken his name by putting his theorem

$$\mathbf{P}(A|B) = \frac{\mathbf{P}(B|A)\mathbf{P}(A)}{\mathbf{P}(B)}$$

at the center of its paradigm. After two centuries of developments and changes, Bayesian statistics is now a considerable force in the modeling of random phenomenon and the analysis of complex data. With this issue we celebrate the good health of Bayesian statistics and, more importantly, point out the directions of development of this self-contained theory of statistics.

One issue worth mentioning is that, in its recent evolution, the pressure of practitioners of statistics and users of Bayesian methodology has clearly modified the approach to Bayesian statistics toward a less dogmatic perspective and pushed forward a stronger unification. While a completely unified theory is clearly impossible, as discussed in the foundational paper of Phil Dawid, some important developments have occurred in the past years that explain why the Bayesian and the Fisher–Neyman–Pearson approaches to testing hypotheses differ so much, and how they could reconcile from a certain (sequential) perspective. This latter

Christian P. Robert is Professor of Statistics, CEREMADE, Université Paris Dauphine, Paris, France (e-mail: xian@ceremade.dauphine.fr). George Casella, Executive Editor of Statistical Science, is Distinguished Professor and Chair, Department of Statistics, University of Florida, Gainesville, Florida 32611–8545, USA (e-mail: casella@stat.ufl.edu).

point is illustrated by Susie Bayarri and Jim Berger. A similar trend can be observed for model choice and the incorporation of non-Bayesian criteria (such as Akaike's) as shown by Ed George and co-authors, with his paper with Merlise Clyde focusing on the problem of variable selection and model averaging.

The evolution of the range of Bayesian inference is clearly visible in nonparametrics, which seemed to be a frequentist *chasse gardée* till recent years. As shown in the papers by both Peter Müller and Fernando Quintana, and by Stephen Walker, the Bayesian approach can provide a more than satisfactory answer to the problem of modeling datasets without parametric prior assumptions.

Besides numerous theoretical developments in the past thirty years, and, arguably, the fundamental logic inherent to Bayesian inference (the inverse problem of conditioning on the result and evaluating causes from effects), another reason for the remarkable rise in the use of Bayesian techniques is the explosion in computational power that occurred at the end of the 1980s with the appearance of MCMC techniques. While (then) powerful personal computers were already available at the end of the 1970s, the existing simulation or numerical techniques were not able to handle the high-dimensional models found in hierarchical Bayesian analysis beyond toy examples. (At that time, importance sampling was still in its infancy and naïve importance functions were unable to face the challenge.) The paper by Andrieu, Douce and Robert describes the sudden impact on Bayesian analysis offered by MCMC techniques, and stresses the complementary impact this had on simulation technology. For example, the papers by Mike Titterington and Michael Jordan are an illustration of the dissolving of the frontier between Statistics and Computer Science. Complex structures like neural networks and other graphical models, that were somehow on the "wrong" side of the barrier, were incorporated into Bayesian statistics. Both papers also incorporate descriptions of variational methods which, although quite natural from a model choice perspective, have not yet met full recognition in the statistical community.