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Comment: On Random Scan Gibbs Samplers

Richard A. Levine and George Casella

1. INTRODUCTION

We congratulate the authors on a review of convergence rates for Gibbs sampling routines. Their combined work on studying convergence rates via orthogonal polynomials in the present paper under discussion (which we will denote as DKSC from here onward), via coupling in Diaconis, Khare and Saloff-Coste (2006), and for multivariate samplers in Khare and Zhou (2008), enhances the toolbox of theoretical convergence analysis. This has the potential of opening new avenues of pursuit for gauging chain convergence in practice, and optimally implementing Gibbs sampler strategies. In this discussion, we focus on the latter, within the context of the random scan Gibbs sampler presented in DKSC. Although the analysis in DKSC does not seem to extend to the random scan implementation we consider, a study of convergence rate and estimator precision is possible, in theory, for special cases as well as in general practice. Our aim is to motivate further research within the context of DKSC to identify objective criteria for optimizing implementation of the random scan Gibbs sampler.

2. REVISITING RANDOM SCAN GIBBS SAMPLERS

The random scan Gibbs sampler considered in DKSC has an equal likelihood of visiting each coordinate, (x, θ), during an iteration of the sampler. As put forth by the seminal convergence theory work of Liu, Wong and Kong (1995) and discussed more recently by Levine and Casella (2006), an optimal implementation of the random scan strategy may visit less often components with a marginal that is easier to understand or describe. For example, in the bivariate cases of DKSC, each iteration of the random scan visits x with probability α_1 and θ with probability $1 - \alpha_1$, where $\alpha_1 \in (0, 1)$, not necessarily equal to 0.5. For the general multivariate problem of sampling a d-vector $\mathbf{X}$, the random sweep strategy is characterized by selection probabilities $\boldsymbol{\alpha} = (\alpha_1, \alpha_2, \ldots, \alpha_d)$, where $\sum_{i=1}^d \alpha_i = 1$, α_i not necessarily equal to $1/d$ for all i.

In the notation of DKSC, the transition kernel of the random scan Gibbs sampler for a function $g \in L^2(P)$ is

$$\begin{aligned} \bar{K} g(x, \theta) &= \alpha_1 \int_\Theta g(x, \theta')\pi(\theta'|x)\pi(d\theta') \\ &\quad + (1 - \alpha_1) \int_{\mathcal{X}} g(x', \theta) f_\theta(x')\mu(dx'). \end{aligned} \tag{1}$$

Unfortunately, $\bar{K}$ in (1) is not readily diagonalizable as the decomposition in the proof of DKSC Theorem 3.1, part (c), relies on the equal selection probabilities ($\alpha_1 = 0.5$) to partition the transition kernel acting on appropriate functions g. However, in the cases of discrete state spaces and Gaussian target distributions, both considered in the exposition of DKSC, we may identify explicit convergence rates and optimally choose selection probabilities. In the following sections, we elaborate on these findings and present an alternative approach with estimator precision as an objective criterion. We also suggest avenues for future research within the context of DKSC to address the random scan Gibbs sampler decision problem.

3. CONVERGENCE RATES

Convergence rates of Gibbs sampling routines may be formulated in two special cases: Gaussian and discrete target distributions. DKSC Section 6.3 eludes to the case of Gaussian distributions, identifying the work of Goodman and Sokal (1989), that shows convergence rates as the largest eigenvalue of a matrix related to the dispersion matrix and an autoregressive transition of the Markov chain (see Khare and Zhou, 2008, as well). Amit (1996) and Roberts and Sahu (2001) provide an alternative expression which lends well to our

Richard A. Levine is Associate Professor, Department of Mathematics and Statistics, San Diego State University, San Diego, CA 92182, USA (e-mail: ralevine@sciences.sdsu.edu). George Casella is Distinguished Professor, Department of Statistics, University of Florida, Gainesville, FL 32611, USA (e-mail: casella@stat.ufl.edu).