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Comment: Demystifying Double Robustness: A Comparison of Alternative Strategies for Estimating a Population Mean from Incomplete Data

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INTRODUCTION

We congratulate Drs. Kang and Schafer (KS henceforth) for a careful and thought-provoking contribution to the literature regarding the so-called "double robustness" property, a topic that still engenders some confusion and disagreement. The authors' approach of focusing on the simplest situation of estimation of the population mean μ of a response y when y is not observed on all subjects according to a missing at random (MAR) mechanism (equivalently, estimation of the mean of a potential outcome in a causal model under the assumption of no unmeasured confounders) is commendable, as the fundamental issues can be explored without the distractions of the messier notation and considerations required in more complicated settings. Indeed, as the article demonstrates, this simple setting is sufficient to highlight a number of key points.

As noted eloquently by Molenberghs (2005), in regard to how such missing data/causal inference problems are best addressed, two "schools" may be identified: the "likelihood-oriented" school and the "weighting-based" school. As we have emphasized previously (Davidian, Tsiatis and Leon, 2005), we prefer to view inference from the vantage point of semiparametric theory, focusing on the assumptions embedded in the statistical models leading to different "types" of estimators (i.e., "likelihood-oriented" or "weighting-based") rather than on the forms of the estimators themselves. In this discussion, we hope to complement the presentation of the authors by elaborating on this point of view.

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Throughout, we use the same notation as in the paper.

SEMIPARAMETRIC THEORY PERSPECTIVE

As demonstrated by Robins, Rotnitzky and Zhao (1994) and Tsiatis (2006), exploiting the relationship between so-called *influence functions* and estimators is a fruitful approach to studying and contrasting the (large-sample) properties of estimators for parameters of interest in a statistical model. We remind the reader that a *statistical model* is a class of densities that could have generated the observed data. Our presentation here is for scalar parameters such as μ, but generalizes readily to vector-valued parameters. If one restricts attention to estimators that are *regular* (i.e., not "pathological"; see Davidian, Tsiatis and Leon, 2005, page 263 and Tsiatis 2006, pages 26–27), then, for a parameter μ in a parametric or semiparametric statistical model, an estimator $\widehat{\mu}$ for μ based on independent and identically distributed observed data z_i, $i=1,\dots,n$, is said to be *asymptotically linear* if it satisfies

$$n^{1/2}(\widehat{\mu}-\mu_0)=n^{-1/2}\sum_{i=1}^{n}\varphi(z_i)+o_p(1) \tag{1}$$

for $\varphi(z)$ with $E\{\varphi(z)\}=0$ and $E\{\varphi^2(z)\}<\infty$, where μ_0 is the true value of μ generating the data, and expectation is with respect to the true distribution of z. The function $\varphi(z)$ is the *influence function* of the estimator $\widehat{\mu}$. A regular, asymptotically linear estimator with influence function $\varphi(z)$ is consistent and asymptotically normal with asymptotic variance $E\{\varphi^2(z)\}$. Thus, there is an inextricable connection between estimators and influence functions in that the asymptotic behavior of an estimator is fully determined by its influence function, so that it suffices to focus on the influence function when discussing an estimator's