

ANOTHER CORRECTION ERROR ESTIMATES FOR BINOMIAL APPROXIMATIONS OF GAME OPTIONS

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BY YAN DOLINSKY AND YURI KIFER

Hebrew University

This note deals with the substantial inaccuracies in Lemmas 3.4 and 3.5 [more specifically, inequalities (4.48) and (4.53) of their proofs] and in Theorems 2.2 and 2.3 of [1] related to the important point that if a game option is not exercised or canceled before the expiration (horizon) time then the seller pays no penalty to the buyer, which is natural but does not agree well with the direct extension of payoff formulas beyond the horizon. The arguments in [1] do not require any modification if penalties in the corresponding game options are extended by zero beyond the horizon which, in view of the Lipschitz-type condition (2.2) there, would be a somewhat restrictive requirement since it eliminates the case of a constant (nonzero) penalty. Of course, there is no problem with the argument there if we consider just the American options case. We will deal with Theorems 2.2 and 2.3 later on in this note (warning the reader that our first correction of the proof there contains inaccuracies) and we start with showing that the estimate of Theorem 2.1 remains true if in place of Lemmas 3.4–3.6 we employ the following argument which extends the idea of Lemma 3.6 there. In the notations, the value of a game option in the Black–Scholes market is given by

$$(1) \quad V(z) = \inf_{\sigma \in \mathcal{T}_{0T}^B} \sup_{\tau \in \mathcal{T}_{0T}^B} E^B Q_z^B(\sigma, \tau)$$

which in view of Lemma 3.3 in [1] should be compared with

$$(2) \quad V_n^{B,\theta}(z) = \inf_{\zeta \in \mathcal{T}_{0,n}^{B,n}} \sup_{\eta \in \mathcal{T}_{0,n}^{B,n}} E^B Q_z^B(\theta_\zeta^{(n)}, \theta_\eta^{(n)}).$$

LEMMA 1. *There exists a constant $C > 0$ such that for all $z, n > 0$,*

$$(3) \quad |V(z) - V_n^{B,\theta}(z)| \leq C(F_0(z) + \Delta_0(z) + z + 1)n^{-1/4}.$$

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