The Annals of Statistics
2006, Vol. 34, No. 2, 1045–1047
DOI: 10.1214/009053606000000182
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CORRECTION

EFFICIENT PARAMETER ESTIMATION FOR SELF-SIMILAR PROCESSES

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The Annals of Statistics (1989) **17** 1749–1766

In the paper [1] the author claimed to have established asymptotic normality and efficiency of the Gaussian maximum likelihood estimator (MLE) for long range dependent processes (with the increments of the self-similar fractional Brownian motion as a special case—hence, the title of the paper). The case considered in the paper was Gaussian stationary sequences with spectral densities $f_\theta(x) \sim |x|^{-\alpha(\theta)} L_\theta(x)$, where $0 < \alpha(\theta) < 1$, $\theta \in \Theta \subset \mathbb{R}^p$ with Θ compact, and $L_\theta(x)$ being a slowly varying function at 0, a typical example being $f_\theta(x) = |x|^{-2d} h_\theta(x)$ and $\theta = (d, \theta_1, \dots, \theta_{p-1})'$. The results were derived under a set of conditions termed (A0)–(A9), where in particular,

(A9) α is assumed to be continuous. Furthermore, there exists a constant C with

$$|f_\theta(x) - f_{\theta'}(x)| \le C|\theta - \theta'| f_{\theta'}(x) \tag{1}$$

uniformly for all x and all θ, θ' with $\alpha(\theta) \le \alpha(\theta')$, where $|\cdot|$ denotes the Euclidean norm.

Unfortunately, this condition rules out long range dependent processes: Consider the case $f_\theta(x) = |x|^{-2d} h_\theta(x)$, $\theta = (d, \theta_1, \dots, \theta_{p-1})'$, with $h_\theta(x)$ being bounded from above and below. Suppose θ' is fixed and $\theta = \theta_n$ with $\theta_n \to \theta'$, where $d = d_n = d' - 1/(2n)$. Then it is easy to verify that (A9) is not fulfilled for $x = 1/(n^n)$ and n sufficiently large.

Luckily (A9) can be relaxed to

(A9′) α is assumed to be continuous.

We now briefly sketch how the results of the paper follow under this weaker condition. We assume that the reader is familiar with the notation and the results of the original paper.

REMARK. If $f_\theta(x) = |x|^{-\alpha(\theta)} h_\theta(x)$, the smoothness conditions in (A0)–(A8) on $f_\theta(x)$ are only fulfilled if α is three times differentiable [which is true, e.g., if $\alpha(\theta) = \theta_1$]. However, formally we need no such assumption since α appears only in upper bounds.

Received October 2004; revised April 2005.