Defining q as the set of conditional distributions

$$\{(P(\Theta = 0 \mid X = x) = 2/3,\ P(\Theta = 1 \mid X = x) = 1/3)): x = 1, 2, 3, \ldots\},$$

Kadane, Schervish, and Seidenfeld (1985) assert that it is "reasonable to claim that q is the posterior for θ given X once finite additivity is accepted," and that the example "makes clear the need for a less restrictive definition of posterior distribution that will allow inference even when a probability cannot be made conglomerable in a specific partition." For q,

$$(2) \qquad \frac{P(\Theta = 1 \mid X = x)}{P(\Theta = 0 \mid X = x)} = 1/2, \quad x = 1, 2, \ldots.$$

To ignore the conflict between (1) and (2), on the grounds that this is merely an expression of acceptable nonconglomerability, is to turn a blind eye to the problem that it raises in the use of P to approximate honest opinion about what odds to quote for Θ given $12 < X < 12^{144}$.

Of course, not all the applications of finitely noncountably additive probabilities are unattractive. As yet, there appear to be no axioms that will discriminate either the probabilities or the applications that are acceptable. It is not easy to see how the necessary weakening or replacement of monotone continuity might be engineered. There may be one or two clues in the work of Seidenfeld and Schervish (1983). Let us hope that Dr. Fishburn will return once again to the topic, in a survey that will remove the remaining obscurities.

ADDITIONAL REFERENCES

KADANE, J. B., SCHERVISH, M. AND SEIDENFELD, T. (1985). Statistical implications of finitely additive probability. In *Bayesian Inference and Decision Techniques with Applications: Essays in Honor of Bruno de Finetti* (P. K. Goel and A. Zellner, eds.). Elsevier Science Publishers B. V. (North-Holland), Amsterdam.

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Comment

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Most of Professor Fishburn's interesting article treats axiom schemes for the relations *is more probable than* and *is at least as probable as*, and the question of when these schemes lead to a compatible probability measure. There are two other approaches to formulating axioms for probabilities interpreted as degrees of belief. The first is due to de Finetti (1937, 1949) and gives a direct economic interpretation to probability numbers. The second was developed by Cox (1961) and Jaynes to formulate axioms for rational beliefs and for how such beliefs should be modified. Perhaps some readers will be interested in a brief description of these two alternative routes.

One version of the de Finetti theory begins with a function P which assigns a real number $P(A)$ to certain events A. Think of $P(A)$ as your price in dollars for a ticket worth \$1 if A occurs and \$0 if not. You are required to be willing to buy or sell a finite number of tickets on any of the events in a given collection $\mathscr{A}$. (There is no need to assume $\mathscr{A}$ is an algebra.) Then de Finetti shows that you are *coherent* in the sense that you cannot be made a sure loser if and only if P is a finitely additive probability measure (or can be extended to be one if $\mathscr{A}$ is not an algebra). An advantage of this approach is that the conditional probability $P(A \mid B)$ can be defined directly as the price of a \$1 ticket on A with the provision that the transaction is called off if B does not occur. A requirement of coherence for these conditional transactions leads to the formula

$$P(AB) = P(B)P(A \mid B)$$

which in turn implies the finite form of Bayes' formula given in Section 7. All of this is explained in detail by de Finetti (1949). There are extensions of de Finetti's result which yield Bayes' formula for infinite partitions (cf. Heath and Sudderth (1978) and Lane and Sudderth (1984)). These extensions involve a strengthening of the coherence condition which is not acceptable to all of de Finetti's followers.

In the Cox-Jaynes theory it is assumed that the *plausibility* of A on the evidence B can be represented by a real number $(A \mid B)$. Qualitative arguments are given for a postulate stating that the plausibility number $(AB \mid C)$ should be some function F of $(B \mid C)$ and $(A \mid BC)$. Because AB is the same as BA, the function F is required to give the same answer if its arguments

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