To get a lower bound on the second term in (A.1), we use the fact that $\inf(R_{**}^{-1}) = \| R_{**} \|^{-1}$, from which it follows that

$$\begin{aligned} \| R_{**}^{-1} t_{*p} \| &\geq \inf(R_{**}^{-1}) \| r_{*p} \| \\ &= \| R_{**} \|^{-1} \| r_{*p} \| \\ &\geq \| R_{**} \|_F^{-1} \| r_{*p} \|. \end{aligned}$$

Since the columns of R_{**} have norm one, $\| R_{**} \|_F^2 = p - 1$ and $\kappa_p^{-2} = \rho_{pp}^2 = 1 - \| r_{*p} \|^2$. Hence

$$\text{(A.3)} \qquad \| R_{**}^{-1} r_{*p} \|^2 \geq \frac{1 - \kappa_p^2}{p - 1}.$$

Combining (A.1), (A.2), and (A.3) we get

$$(p - 1)\max_{i \neq j} \kappa_i^2 \geq \sum_{i \neq j} \kappa_i^2 \geq p - 1 + \frac{\kappa_p^2 - 1}{p - 1},$$

which is equivalent to (4.4).

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Comment

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Statisticians and numerical analysts owe a large debt of gratitude to Dr. Stewart for his demonstration and lucid exposition of the mathematical connection between the condition number and the parameter variance inflation factors. In doing so, he has also clarified the reasons why the condition number is not really helpful in the multiple regression context, nor in many other contexts. The insights he provides in this paper are important for all statisticians, because collinearity problems occur in many statistical contexts, including multiple linear regression, nonlinear regression, unbalanced analysis of variance, and estimation from inverse integral transform models. In this brief commentary I have selected three facets of Dr. Stewart's paper for discussion.

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