

Spiegelhalter cites Zadeh in support of the view that Dempster's rule of combination can lead to unintuitive results. For a reply to Zadeh's arguments, see Shafer (1986a).

The Bishop of Bath and Wells whose work on probability Lindley discusses was named George Hooper. Hooper actually became a bishop only in 1703, long after his work on probability was published. Details about Hooper's life and work are given by Grier (1981). Hooper gave two rules for combining testimony, a rule for concurrent testimony and a rule for successive testimony. I have discussed these rules and their Bayesian counterparts elsewhere (Shafer, 1978, 1986c).

Hooper's rules were widely admired in the 18th century; they appear, for example, in Diderot's *Encyclopedie*. The Bayesian analysis that Lindley reviews, together with a corresponding analysis for the case of successive testimony, displaced Hooper's rules in the early 19th century (see Shafer, 1978). But this Bayesian account of "the probability of testimony" quickly became a laughingstock. It was roundly and justly denounced both by logicians critical of probability, such as John Stuart Mill, and by probabilists who preferred a frequentist interpretation, such as Antoine-Augustin Cournot.

Rejoinder

Dennis V. Lindley

I find myself in general agreement with the contributions of Watson and Spiegelhalter. Watson is right when he says we do not have to accept Savage's axioms. But it is desirable to have an axiom system to support one's calculations and the lack of them must count against the alternatives to probability. Spiegelhalter is right when he says that ultimately it's the appeal of probability that matters: people will see that it makes good sense. Just as with Euclidean geometry, it is the operational aspect that counts, rather than Euclid. Watson queries the existence of the Great Scorer. I do not think it matters because one would wish to behave in such a way that one could not be exposed by his or her arrival. I would regard it as a serious proposal to pay meteorologists, or even medical doctors, according to their scores.

Whilst I find myself in dispute with Shafer, his arguments command respect and are not easily refuted. He contends that the axioms depend on conditional probability and expected utility, rather than

The theory of belief functions does not require us to go back to Hooper's rules. Instead it provides a framework that includes both Hooper's analyses and the Bayesian analyses as special cases, along with many intermediate possibilities. The virtue of this flexibility is that we can tailor our analysis to our actual evidence. If we have significant prior evidence, we can use it. If we have evidence for causal dependence between the witnesses, we can use it. If we have instead evidence for dependence in our uncertainties about the witnesses, we can use it. By relating the numbers we offer to actual evidence in this way, we can hope to escape the ridicule that so wounded subjective probability in the 19th century.

ADDITIONAL REFERENCES

- GRIER, B. (1981). George Hooper and the early theory of testimony. Dept. Psychology, Northern Illinois Univ.
- KONG, A. (1986). Multivariate belief functions and graphical models. Ph.D. dissertation, Dept. Statistics, Harvard Univ.
- SHAHER, G. (1978). Nonadditive probabilities in the work of Bernoulli and Lambert. *Arch. Hist. Exact Sci.* **19** 309-370.
- SHAHER, G. (1986c). The combination of evidence. *Internat. J. Intelligent Systems* **1** 155-179.
- SHAHER, G., SHENOY, P. and MELLOULI, K. (1986). Propagating belief functions in qualitative Markov trees. Working paper no. 186, School of Business, Univ. Kansas.

that these depend on the axioms. While it is true that historically the concepts pre-date any axiom system, Savage introduced the axioms in order to justify a system, classical statistics, that denies conditional probability (of a hypothesis) and does not admit expected utility (with an expectation over unknowns); and he was much surprised when the axioms destroyed that system.

The scoring-rule argument works for almost every rule and does not depend on 0 or 1 as Shafer suggests. The preferences in Bayesian decision analysis are not necessarily sharp. If d_1 has expected utility 10.927 and d_2 10.926, then d_1 is preferred only slightly to d_2 . The analysis is designed to select *an* act because only one act is typically possible.

Shafer also raises the issue of constructive probability. It is difficult, having experienced A_1 , to think of probabilities for A_1 if only because probability describes uncertainty and A_1 is no longer uncertain. My response is that we should try to develop methods that