

## ADDITIONAL REFERENCES

- BARNDORFF-NIELSEN, O. E. and HALL, P. (1988). On the level-error after Bartlett adjustment of the likelihood ratio statistic. *Biometrika* **75** 374–378.
- BOLTHAUSEN, E. (1986). Laplace approximations for sums of independent random vectors. *Probab. Theory Related Fields* **72** 305–318.
- ELLIS, R. S. (1985). *Entropy, Large Deviations, and Statistical Mechanics*. Springer, New York.
- FÖLLMER, H. (1987). Random fields and diffusion processes. *École d'Été de Probabilités de Saint Flour XVI—1986. Lecture Notes in Math*. Springer, Berlin. To appear.
- GROENEBOOM, P. and OOSTERHOFF, J. (1981). Bahadur efficiency and small-sample efficiency. *Internat. Statist. Rev.* **49** 127–141.
- KHINCHIN, A. I. (1949). *Mathematical Foundations of Statistical Mechanics*. Dover, New York.
- STROOCK, D. W. (1984). *An Introduction to the Theory of Large Deviations*. Springer, New York.

# Comment

H. E. Daniels

It is a pleasure to congratulate Nancy Reid on her masterly review that will be much cited by present and future statisticians. Indeed, it is so comprehensive that it leaves one little room for comment. However, I might venture to make one or two remarks.

Statisticians have naturally felt more at home with the conjugate density approach to the saddlepoint approximation, but the appearance of Lugannani and Rice's formula for tail probabilities has forced them to face up to the complex variable approach for which there seems no alternative in that case. As might be expected, applied probabilists have been more at ease with analytic methods. Borovkov and his coworkers, starting with his basic papers (1960) on the maxima of sums of random variables, have used the saddlepoint method systematically in applications to boundary crossing problems and queuing theory. Not being concerned with inference as such, the work is understandably not quoted in the present review. Again, in Chapter 5 of their well known book "Queues," Cox and Smith (1961) indicated the value of the method for approximating to the distribution of busy periods and similar quantities.

Turning now to the review itself, perhaps too much can be made of the difficulty of computing the

cumulant generating function (see comments in Sections 5 and 6). Given a knowledge of the basic density, the computation of the cumulant generating function and its derivatives can be routinely carried out without having to know its analytic form. In fact, it is because one can do the same thing with the empirical distribution function that Davison and Hinkley's successful application of saddlepoint methods to bootstrapping is possible. With even moderate computing facilities such difficulties lose their importance.

My final remark relates to the work on bootstrapping just referred to. Davison and Hinkley show convincingly that saddlepoint approximations reflect accurately the results obtained by simulation, with an enormous saving in computer time, particularly when considering conditional distributions. However, there remains a nagging doubt when the data set is of moderate size. Because one is sampling from a distribution with finite support when the underlying distribution is, for example, normal, there may be a bias in the bootstrap or saddlepoint estimates near the tails. My colleague G. A. Young and I are currently looking into this bias using a combination of saddlepoint and simulation techniques and hope to report soon on our findings.

## ADDITIONAL REFERENCES

*H. E. Daniels is retired from the University of Birmingham. His mailing address is: Statistical Laboratory, Department of Pure Mathematics and Mathematical Statistics, University of Cambridge, 16 Mill Lane, Cambridge CB2 1SB, United Kingdom.*

- BOROVKOV, A. A. (1960). Limit theorems on the distribution of maxima of sums of bounded lattice random variables. I, II. *Theory Probab. Appl.* **5** 125–155, 341–355.
- COX, D. R. and SMITH, W. L. (1961). *Queues*. Methuen, New York.