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Comment

Trevor Hastie and Robert Tibshirani

Professor Ramsay has written an informative paper about a topic that is new (at least to us) and deserves exposure. The techniques that he describes and his software implementations are potentially useful in a number of different areas. However, we found that after careful reading of the paper and experimenting with monotone splines, we are in substantial disagreement with him over a number of important points. In particular:

- The monotonicity assumption inherent in monotone splines will sometimes (often?) be unwarranted. A more useful modeling technique allows a choice of smoother for each variable, perhaps between linear, monotone and nonmonotone, together with a strategy for selecting the appropriate form. A general estimation procedure called *backfitting* can be used to estimate models of this kind.
- The number and position of knots *can* make a difference and we can see no clear way to make these choices. Other smoothing techniques such as smoothing splines have the significant advantage that a single smoothing parameter controls the smoothness of the output.
- The number of parameters inherent in a monotone spline is *not* "far fewer" than the number in a cubic smoothing spline or other common smoothers, given a comparable amount of smoothness.
- The data analysis in the paper are somewhat weak and potentially misleading.

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