I studied statistically simulated Poisson Dirichlet tessellations, in particular the point process of vertices of cells. Surprisingly we found that the corresponding second-order product density $\rho(r)$ has a striking form: it seems to be true that

$$\lim_{r\to 0} \rho(r) = \infty,$$

or, at least, $\rho(0)$ seems to be very great. Usually, such behavior of a product density is an indicator of a high degree of clustering. By visual inspection of some simulated tessellations we found that clusters of vertices in the usual sense of the word are not typical for these tessellations, but there appear frequently very short edges (of otherwise "normal" cells) or pairs of vertices very close together.

With respect to statistical shape problems related to "landmarks" in the sense of Bookstein (1978, 1986), I should like to ask the following question. Imagine three nonintersecting circles in the plane. Take a random point in each of the circles, for example uniformly or with respect to any distribution. Form the triangle having the three points as their vertices. Is it possible to give the corresponding shape density?

ADDITIONAL REFERENCES

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Rejoinder

David G. Kendall

It is appropriate that Professor Bookstein should open this discussion in view of the importance of his work and the great influence that this has had through his own presentation in *Statistical Science* and his earlier 1978 monograph. I was already deeply involved in shape theory when I first read the latter, but did not at that time foresee how closely our two different and differently motivated approaches would converge. It is all the more valuable, therefore, that he has generously taken the time and trouble to survey their current interactions and differences of emphasis. His remarks will deserve careful study.

Professor Small's contribution is full of wise insights, and novel suggestions are made that I shall think about deeply. "Projection-pursuit" viewing of higher dimensional shape manifolds may well be a reality a few years from now. My current practice, not so technologically ambitious, is to try to understand these spaces as thoroughly as possible, and then to seek dimension-lowering projections that retain the important information and make it visible in a helpful way. One example of such a procedure will be found in my contribution to the discussion on Bookstein's 1986 paper referred to above. Of course I agree with the remarks that he and others have made about the advantages of having a variety of visual displays available. I recall that Kipling wrote a fine poem on a similar topic many years ago.

Professor Mardia's contribution was a shock to me because I did not expect to see so beautiful a solution as that found by Mardia and Dryden to the important problem they have studied. It makes one ask, why is it so beautiful? What has happened to all the horrible noncentral χ^2's? Of course the Gaussian distribution never ceases to spring surprises on us. I discussed Mardia's remarks with Wilfrid Kendall, and it occurred to us that a dynamic approach might at least "explain" what lies behind such a nice formula. So here are a few remarks intended only to illuminate the anatomy of the problem.

To start with it will be necessary to change the notation a little. We identify Mardia's κ with $s_0^2/(4c^2t)$, where c is a diffusion constant, t is the time elapsed during the interval considered and s_0 is a linear measure of the size of the triangle $\Delta_0 = (A_0, B_0, C_0)$ at the beginning of that time interval. The Mardia-Dryden formula then gives the law of distribution of the shape at the end of the time interval when we know what the shape was to start with. Notice that in this formulation it is no longer necessary to exclude $A_0 = B_0 = C_0$ as a possible initial shape, for then $s_0 = 0$, and this makes $\kappa = 0$, and then the Mardia-Dryden formula tells us that the distribution of size at the end of the interval is uniform over the sphere, as it ought to be.

More generally let us write $\zeta(t)$ for the shape of $\Delta_t = (A_t, B_t, C_t)$ at time t, this being undefined at