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Rejoinder

Robert E. Kass

I am very grateful to the discussants for their comments, which have substantially enriched the material presented here. The remarks of Professors Amari, Barndorff-Nielsen, and Reid and Fraser require no reply. I do, however, wish to answer the specific queries raised by Professors Bernardo and Rao.

With regard to Rao's query (1), concerning characterizations of the information metric, I would refer interested readers to the original work of Centsov (1972) and the newer work of Picard (1989). I am not sure what Rao has in mind in his query (2) about the choice of affine connection. Part of the answer may come from the results of Centsov and Picard, but if Professor Rao is referring to the choice of α in the α -connection, perhaps helpful to the intuition is the observation in Kass (1984) that vanishing of the α -connection coefficients when $\alpha = -1, -\frac{1}{3}, 0, 1$ occurs for the bias-reducing, skewness-reducing, variance-stabilizing, and natural parameterizations, respectively, and when $\alpha = \frac{1}{3}$, it occurs for the parameterization in which the expected values of the third derivatives of the loglikelihood vanish. These parameterizations were characterized in differential equation form, in the one-parameter case, by Hougaard (1982). There is also a very nice answer to part of Rao's query (3), due to Amari (1985, 1987a). In brief, Amari used higher derivatives of the imbedding $\eta(\cdot)$, defining a curved exponential family, to define both higher-order curvatures and appropriate statistics based on higher-order derivatives of the loglikelihood function. With these he obtained a complete decomposition of the information in the sample as an asymptotic expansion with geometrically-interpretable terms of decreasing order associated with the loglikelihood derivatives.

Finally, Professor Rao's point (4), and Professor Bernardo's request for comments in the rejoinder, concern Jeffreys' general rule for choosing a prior. I have a few things to say about this, though for the sake of brevity I will not try to argue my opinions in detail.

As a preliminary remark, I emphasize that by "reference prior" I mean a prior chosen according to any formal rule that may be applied without detailed consideration of the data-analytic context. Such a prior need not be considered "noninformative" in any well-defined sense. This is an important point, since it is dubious that the concepts of ignorance and lack of information can be given satisfactory definitions. I believe the idea of selecting a prior by *convention*, as a "standard of reference," analogous to choosing a standard of reference in other scientific settings, is due to Jeffreys (1955) page 277. This notion and terminology was adopted by Box and Tiao (1973) page 23. Unfortunately, Bernardo (1979) used the term "reference prior" for a specific rule, rather than the general concept, and this occasionally causes confusion.

There is great convenience in conventional choices, throughout statistics and throughout science. But convenience should not be confused with necessity: one might say that conventions are useful as long as they are not taken too seriously. Thus, I see the convenience in reference priors, just as I recognize the convenience in conventional levels of significance. In applications, however, such conveniences must be questioned. Sometimes they are justifiable time-savers, especially for communicating results, but often they are not. I consider reference priors to be "default" choices, but they are to be used only when their