difficulty. Indeed the reference prior for the ordered sequence $(\theta_1, \cdots, \theta_m)$ is (see Berger and Bernardo, 1989 for details)

$$\begin{aligned}\pi_R(\theta_1, \cdots, \theta_m) &= \pi(\theta_1)\pi(\theta_2 \mid \theta_1) \cdots \pi(\theta_m \mid \theta_1, \theta_2, \cdots, \theta_{m-1}) \\ &= (\pi^{-m}) \prod_{i=1}^{m} [\theta_i^{-1/2}(1 - \textstyle\sum_{j=1}^{i} \theta_j)^{-1/2}],\end{aligned}$$

and the corresponding marginal reference distribution for θ_1 is

$$\begin{aligned}\pi_R(\theta_1 \mid y_1, \cdots, y_m, n) &= \pi_R(\theta_1 \mid y_1, n) \\ &= Be(\theta_1 \mid y_1 + 1/2, n - y_1 + 1/2),\end{aligned}$$

no matter how many cells are considered.

ADDITIONAL REFERENCES

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Comment

C. R. Rao

Geometric ideas do help in suggesting intuitive solutions to some complex problems and also in obtaining explicit solutions to specific problems through geometric methods. In his paper, "The geometry of asymptotic inference," Dr. Kass has demonstrated these two aspects by providing us with an excellent review of the past work and presenting some new ideas on the use of differential geometry in interpreting and developing statistical methodology. As Dr. Kass observed, differential geometry is a branch of mathematics "which is largely unfamiliar to most statisticians and may seem rather technical." I hope his paper will create some interest and encourage research in the differential geometric approach to statistical problems. However, I am tempted to share the caution expressed by Dr. D. J. Finney, in a similar situation, referring to some recent papers in multivariate analysis: "Amongst the many papers on statistical science published today, some appear to find outlets to mathematical theory without materially assisting scientific research." One may not fully subscribe to Dr. Finney's view, but the message is clear that enrichment of statistical methodology can take place only if its development is motivated by practical problems that are formulated in statistical terms. In this process, sophisticated mathematics could be used. I hope and believe as Dr. Kass does, that although "no claim can be made as yet that differential geometric research has made inroads into a large class of problems that is otherwise unreachable, the methods are so powerful, and the connections with statistics so plausible, that some further developments, of great methodological importance, might well occur."

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In introducing differential geometric methods in statistics, I was motivated by the problem of discrimination between "populations" or "probability distributions " (p.d.'s), which naturally led to the need to introduce a metric in the space of p.d.'s. With a distance defined between two p.d.'s, it is possible to study the configuration of a given set of p.d.'s in terms of clusters and their hierarchical relationships.

In the case of a parametric family of p.d.'s characterized by a set of densities $\{f(x, \boldsymbol{\theta}): \boldsymbol{\theta} \in \Theta\}$, the metric was introduced by furnishing the parameter space Θ with a Riemannian quadratic differential metric (QDM)

$$(1) \qquad \sum g_{ij} d\theta_i d\theta_j$$

where $\boldsymbol{\theta} = (\theta_1, \theta_2, \cdots)'$, and (g_{ij}) is the Fisher information matrix (see Rao, 1945).

Using the QDM, one can compute the geodesic distance between any two p.d.'s represented by any two parameters θ and ϕ, which we denote by $D_g(\theta, \phi)$.