becomes a key tool for connecting information theory and statistics.

Linear Programming Problem

It is interesting that the dual geometry is useful for some other problems. When a convex function $\Psi(\theta)$ is defined, we have a Legendre transformation from θ to η with a dual convex function $\Phi(\eta)$. We can introduce a dually flat geometry when it is equipped with a pair of convex functions. In the case of statistics, Φ is the negative of the entropy function and Ψ is the cumulant generating function. We have natural convex functions derived from linear and non-linear programming problems.

It is interesting to point out that a continuous version of the Karmarkar inner method is just to proceed along an m-geodesic in the space thus equipped with the dual connections. This method can easily be generalized to a nonlinear programming problem. This shows a wide applicability and universality of dual geometry.

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Comment

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Dr. Kass' fine account calls for little comment in itself. However, as he himself stresses, it leaves out parts of the subject, particularly of the more advanced aspects, and it may be useful here to outline briefly some of these parts so as to provide the interested reader with a fuller, though still far from comprehensive, picture of the scene. The discussion below relates mainly to work with which I have been to some degree associated, and it gives, in particular, virtually no impression of the important and extensive work of S.-I. Amari and his collaborators.

As will be indicated, the statistical problems have led to various developments and questions of a purely mathematical nature, and there are also interesting relations to theoretical physics.

INDEX NOTATION

The index notation of classical differential geometry and certain extensions thereof have turned out to be highly useful for many calculations in statistics, including some that are not of differential geometric nature (cf. McCullagh, 1987; Barndorff-Nielsen and Blæsild, 1988b; Barndorff-Nielsen and Cox, 1989, Chapter 5). The index notation makes many multivariate calculations just as easy as the corresponding

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