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Comment

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del Pino is to be congratulated for his extensive survey of iterative least squares methods. In particular, I welcome the emphasis on the parallel between statistical properties of the model and the structure of the algorithm. This is where statistical computing distinguishes itself from the general area of optimization. An example of this is the role of orthogonality of parameters (cf. Cox and Reid, 1987), which implies an exact or approximate block diagonal structure of the Hessian of the log-likelihood function, with consequent simplification of the calculations. Another example is the discussion in Jørgensen (1984) of marginal and conditional maximum likelihood calculations.

Actually, I think that this marriage between algorithms and statistical theory will be taken much further in the future, and while, at the moment, iterative weighted least squares algorithms are probably the best general class of statistical algorithms available, I predict that the use of iterative least-squares methods will soon be changing. One of the driving forces in this development is the theory related to Barndorff-Nielsen's formula (cf. Barndorff-Nielsen, 1988; Reid, 1988 and references therein) and associated methods, such as saddlepoint approximations, modified profile likelihoods and so on. It is possible that these developments, in particular their geometric aspects, will lead to new and improved statistical algorithms.

To illustrate the potential influence of statistical theory on computing habits, consider the fact that the iterative weighted least-squares algorithm effectively ignores the second derivative of the model function h , denoted $E(\beta)$ by del Pino. On the other hand the

theory associated with Barndorff-Nielsen's formula is effectively the systematic exploration of high-order derivatives of the likelihood, which certainly involves quantities such as $E(\beta)$. Hence, the advantage of iterative weighted least-squares methods, that $E(\beta)$ need not be calculated, will soon become unimportant, because $E(\beta)$ is needed for other purposes. In conclusion, statistical calculations involve much more than just the maximization of the likelihood or of some other objective function, and future statistical computer systems will to a larger extent than is the case today, involve a complete system of procedures for answering various types of inferential problems concerning the data. No doubt, automatic execution of symbolic mathematical calculations will play a crucial role in these developments.

In the meantime, I would like to mention some aspects of iterative weighted least-squares methods considered in Jørgensen (1984). There, I considered what I call the delta algorithm, which is nothing more than the iterative weighted least-squares algorithm with a general A -matrix, concentrating mainly on the case of a separable structure for the likelihood, and the possibility for implementing the algorithm in GLIM. The paper discussed the relation with various other algorithms and mentioned the algorithm for robust estimation considered by del Pino in connection with (3.10), which I referred to as the case of "score weights." In fact, this algorithm may be used in connection with any objective function and is not specific to robust estimation. Among other choices for A considered in Jørgensen (1984) was the case (referred to as "deviance weights"), which, in the language of generalized linear models, corresponds to a data-dependent link function, such that the objective function g becomes exactly quadratic. In other words, all the nonlinearity of the model is "thrown" into the link function. The point here is that there exists a

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