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Rejoinder

David Pollard

I find myself in the position of a man who has just pointed out how one can balance a checkbook using a high-powered graphics workstation. Professor Dudley responds by suggesting some further applications in the same spirit. Professors Giné and Zinn point out that one can also use the machine for high speed interactive graphics. Professor Pyke mentions other uses more suited for a piece of high technology, while suggesting (perhaps tongue in cheek) that my particular checkbook might also be balanced using a hand-held calculator. Professors Csörgő and Horváth demonstrate that their super parallel processor can also balance checkbooks.

In large part I agree with, and welcome, the comments of this distinguished group of discussants. But to maintain the correct atmosphere of contrariness and provocation, I will find some way to disagree with all of them.

Professor Dudley suggests that Fréchet differentiability, with the right choice of norm, should be used in preference to compact differentiability. As he has convincingly argued in his 1989 preprint, this new viewpoint does free Fréchet differentiability from the uncomfortable constraint of distribution functions on the real line. However, compact differentiability (with derivative Δ_x) of a functional T is enough to imply

$$\sqrt{n}\,[T(x + z_n/\sqrt{n}) - T(x)] = \Delta_x \cdot z_n + o(1)$$

for each convergent sequence $\{z_n\}$, a property that is ideally suited to application of Dudley's (1985) almost uniform representation theorem. Gill (1987) has explored this aspect of compact differentiability.

Dudley also suggests substitution of the smooth convex $\rho(x)$ for $|x|$, to eliminate the problems caused by nondifferentiability of $|x|$ at the origin. As a device to simplify the asymptotic theory this is unnecessary (Pollard 1989a); Tchebychev's inequality, the CLT for bounded (vector-valued) summands, and an elementary convexity argument can handle the estimator, even for $c = 0$.

Professors Giné and Zinn quite properly point out some of the beautiful general theory—in particular, the work of Talagrand—that I failed to mention. I feel that conditions expressed in terms of limiting Gaussian processes will not appeal to many potential users of empirical process theory, even though there are excellent theoretical reasons for preferring their approach. At this stage in the history of the world, I feel it is more important that potential users be enticed by small examples of empirical process ideas rather than be impressed and intimidated by the full force and elegance of the latest theory. Times will change. More papers along the lines of Giné and Zinn (1988) will convince us all that sample path properties of abstract Gaussian processes are relevant, even for popular topics such as the bootstrap.

Jain and Marcus (1975, inequality 2.30) did use the idea of dominating a process involving Rademachers by a related Gaussian process, but Giné and Zinn are right concerning the role of the inequality in the